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An improved multi-objective population-based extremal optimization algorithm with polynomial mutation

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ABSTRACT

As a recently developed evolutionary algorithm inspired by far-from-equilibrium dynamics of self-organized criticality, extremal optimization (EO) has successfully applied to a variety of benchmark and engineering optimization problems. However, there are only few reported research works concerning the applications of EO in the field of multi-objective optimization. This paper presents an improved multi-objective population-based EO algorithm with polynomial mutation called IMOPEO-PLM to solve multi-objective optimization problems (MOPs). Unlike the previous multi-objective versions based on EO, the proposed IMOPEO-PLM adopts population-based iterated optimization, a more effective mutation operation called polynomial mutation, and a novel and more effective mechanism of generating new population. From the design perspective of multi-objective evolutionary algorithms (MOEAs), IMOPEO-PLM is relatively simpler than other reported competitive MOEAs due to its fewer adjustable parameters and only mutation operation. Furthermore, the extensive experimental results on some benchmark MOPs show that IMOPEO-PLM performs better than or at least competitive with these reported popular MOEAs, such as MOPEO, MOEO, NSGA-II, A-MOCLPSO, PAES, SPEA, SPEA2, SMS-EMOA, SMPSO, and MOEA/D-DE, by using nonparametric statistical tests, e.g., Kruskal–Wallis test, Mann–Whitney *U* test, Friedman and Quade tests, in terms of some commonly-used quantitative performance metrics, e.g., convergence, diversity (spread), hypervolume, generational distance, inverted generational distance.

1. Introduction

It has been widely recognized that a variety of real-world engineering problems with multiple possible conflicting objectives are formulated as multi-objective optimization problems (MOPs) [1,20,22,53,60]. Unlike single objective optimization problems, MOPs are studied to find a set of satisfied trade-off solutions known as Pareto-optimal solutions. In general, there are two fundamental issues in the design of multi-objective optimization algorithms [11,19,29,39]. The first one is how to improve the convergence performance of the obtained non-dominated solutions, i.e., how to minimize the distance of the obtained non-dominated solutions to the Pareto-optimal front, and the other issue is how to maximize the diversity of the obtained Pareto front.

In practice, the classical mathematical programming methods are difficult to solve these MOPs where the Pareto front is concave or disconnected [19]. Consequently, multi-objective evolutionary algorithms (MOEAs) have attracted the increasing

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interests and considerable attentions of academic and industrial communities [12,29–31,34,40,46–48,52,57–59,64] during the past two decades due to their ability of dealing with a set of possible solutions simultaneously and finding several Pareto-optimal solutions in a single run. This paper presents another novel multi-objective evolutionary algorithm based on extremal optimization (EO) to solve MOPs.

As a novel meta-heuristic optimization algorithm originally inspired by far-from-equilibrium dynamics of self-organized criticality (SOC) [3], extremal optimization (EO) [6,7] provides a novel insight into optimization domain because it merely selects against the bad instead of favoring the good, randomly or according to a power-law probability distribution. The mechanism of EO can be characterized from the perspectives of statistical physics, biological co-evolution and ecosystem [42]. So far, the basic EO algorithm and its modified versions have been successfully applied to a variety of benchmark and real-world engineering optimization problems, such as graph partitioning [8], graph coloring [9], travelling salesman problem [16,62], maximum satisfiability (MAX-SAT) problem [43,63], numerical optimization problems [15,41], community detection in complex network [27], steel production scheduling [17], design of heat pipe [55], and unit commitment problem for power systems [26]. For more comprehensive introduction concerning EO, the readers are referred to the surveys [10,61].

Unfortunately, to the best of our knowledge, there is only few reported research work concerning the applications of EO in the field of multi-objective optimization [13,14,45,54]. Chen and Lu [13] propose an individual elitist $(1 + \lambda)$ multi-objective algorithm called multi-objective extremal optimization (MOEO) based on a single solution, in which a new hybrid mutation operator combining Gaussian mutation with Cauchy mutation to enhances the exploratory capabilities. In [14], another Pareto-based algorithm named Multi-objective Population-based Extremal Optimization (MOPEO), which adopts population-based iterated mechanism and non-uniform mutation [32]. The superiority of MOEO and MOPEO to these competitive algorithms including the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [23], the Pareto Archived Evolution Strategy (PAES) [36], and the Strength Pareto Evolutionary Algorithm (SPEA) [65] is validated by the experimental results on five benchmark functions including ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6. It should be emphasized that the individual-based or population-based iterated mechanism, mutation operation, and mechanism of updating the current population play critical roles in controlling the optimization process and sequentially affecting the performance of any multi-objective evolutionary algorithm based on EO. This paper presents an alternative novel improved multi-objective population-based extremal optimization algorithm with polynomial mutation called IMOPEO-PLM to solve continuous MOPs. The key idea behind IMOPEO-PLM is population-based optimization consisting of the following main operations: generation of random initial population, updating the current population based on an effective polynomial mutation, Pareto-based fitness assignment strategy based on non-dominated sorting, and a novel mechanism of generating new population, updating the external archive according to the archive controller and the crowding-distance metric. Inspired by the research works [25,28,44,50,51] concerning the use of statistical techniques for analyzing evolutionary algorithms' behavior over optimization problems, we compare the proposed IMOPEO-PLM is with other reported competitive MOEAs based on nonparametric statistical tests, e.g., Kruskal–Wallis test, Mann–Whitney U test, Wilcoxon signed ranks test, Friedman and Quade tests, over a variety of benchmark MOPs, e.g., ZDT, DTLZ, WFG problems [31]. These reported MOEAs include MOEO [13], MOPEO [14], NSGA-II [23], PAES [36], SPEA [65], SPEA2 [66], attributed multi-objective comprehensive learning particle swarm optimizer (A-MOCLPSO) [2], SMS-EMOA [4], speed-constrained multi-objective particle swarm optimization (SMPSO) [49], and MOEA/D-DE [38]. Additionally, non-parametric statistical tests used in this paper are based on average commonly-used quantitative performance metrics [33] of at least 30 independent runs for each problem, e.g., convergence, diversity (spread), hypervolume, generational distance, inverted generational distance.

The rest of this paper is organized as follows. In Section 2, we give preliminaries concerning multi-objective optimization problems and EO. Section 3 presents the proposed IMOPEO-PLM algorithm. The experimental results on benchmark MOPs are given and discussed in Section 4. Finally, we give the conclusion and open problems in Section 5.

2. Preliminaries

2.1. Multi-objective optimization problems

Formally, a multi-objective unconstrained minimization problem with n decision variables and m objectives are generally defined as follows [18]:

$$\text{Minimize } F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})) \tag{1}$$

where $\mathbf{x} = (x_1, x_2,\ldots, x_n) \in \Omega$ is the vector of n decision variables, each decision variable x_i is bounded with lower and upper limits $l_i \le x_i \le u_i$, $i = 1, 2, \ldots, n$, F: $\Omega \to R^m$ consists of m real-valued objective functions and R^m is defined the objective space.

An objective vector $\mathbf{u} = (u_1, u_2, \ldots, u_m) \in R^m$ is defined to dominate another objective vector $\mathbf{v} = (v_1, v_2, \ldots, v_m) \in R^m$ (denoted as $\mathbf{u} \prec \mathbf{v}$) if and only if all the following two conditions should be satisfied simultaneously: (1) $\forall i \in \{1, 2, \ldots, m\}, u_i \le v_i$, and (2) $\exists i \in \{1, 2, \ldots, m\}, u_i < v_i$. A decision vector $\mathbf{x} \in \Omega$ is defined to be non-dominated (or Pareto optimal) with respect to Ω if and only if there does not exist another decision vector $\mathbf{x}* \in \Omega$ such that $F(\mathbf{x}^*) \prec F(\mathbf{x})$. The Pareto-optimal set consists of all Pareto optimal solutions in the entire search space and the Pareto-optimal front is defined as the set of all objective functions values corresponding to the Pareto optimal solutions.