Information Sciences 330 (2016) 104–124

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Multicriteria Pythagorean fuzzy decision analysis: A hierarchical QUALIFLEX approach with the closeness index-based ranking methods

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ARTICLE INFO

Article history:
Received 23 May 2015
Revised 21 September 2015
Accepted 6 October 2015
Available online 16 October 2015

Keywords:
Multicriteria decision making
Pythagorean fuzzy number
Interval-valued Pythagorean fuzzy number
The QUALIFLEX approach
The closeness index

ABSTRACT

Pythagorean fuzzy set initially developed by Yager (2014) is a new tool to model imprecise and ambiguous information in multicriteria decision making problems. In this paper, we propose a novel closeness index for Pythagorean fuzzy number (PFN) and also introduce a closeness index-based ranking method for PFNs. Next, we extend the Pythagorean fuzzy set to present the concept of interval-valued Pythagorean fuzzy set (IVPFS) which is parallel to interval-valued intuitionistic fuzzy set. The elements in IVPFS are called interval-valued Pythagorean fuzzy numbers (IVPFNs). We further introduce the basic operations of IVPFNs and investigate their desirable properties. Meanwhile, we also explore the ranking method and the distance measure for IVPFNs. Afterwards, we develop a closeness index-based Pythagorean fuzzy QUALIFLEX method to address hierarchical multicriteria decision making problems within Pythagorean fuzzy environment based on PFNs and IVPFNs. This hierarchical decision problem includes the main-criteria layer and the sub-criteria layer in which the relationships among main-criteria are interdependent, the relationships among sub-criteria are independent and the weights of sub-criteria take the form of IVPFNs. Therefore, in the developed method we first define the concept of concordance/discordance index based on the closeness index-based ranking methods and compute the sub-weighted concordance/discordance indices by employing the weighted averaging aggregation operator based on the closeness indices of IVPFNs. In order to take main-criteria interactions into account, we further employ Choquet integral to calculate the main-weighted concordance/discordance indices. By investigating all possible permutations of alternatives with the level of concordance and discordance of the complete preference order, we finally obtain the optimal rankings of alternatives. The proposed method is implemented in a risk evaluation problem in order to demonstrate its applicability and superiority. The salient features of the proposed method, compared to the state-of-the-art QUALIFLEX-based methods, are: (1) it can take the interactive phenomena among criteria into account; (2) it can manage simultaneously the PFN and IVPFN decision data; (3) it can deal effectively with the hierarchal structure of criteria. The proposed method provides us with a useful way for hierarchical multicriteria decision making problems within Pythagorean fuzzy contexts. In addition, we also extend the proposed method to manage heterogeneous information which includes five different types of information such as real numbers, interval numbers, fuzzy numbers, PFNs and hesitant fuzzy elements.

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http://dx.doi.org/10.1016/j.ins.2015.10.012

1. Introduction

The concept of Pythagorean fuzzy sets (PFSs) originally introduced by Yager [43] is a useful extension of the concept of Atanassov's intuitionistic fuzzy sets (A's IFSs) [3]. The main difference between the two types of fuzzy sets is that the former is required to satisfy the constraint condition that the square sum of the membership degree and the non-membership degree is equal to or less than one but the sum of the two degrees is not required to be less than one, while the latter is required to satisfy the condition that the sum of the two degrees is equal to or less than one. Yager [43] gave an example to illustrate this situation: one expresses his preference about the degree of an alternative A_i in a criterion C_j, and he may give the degree to which the alternative A_i satisfies the criterion C_j as $\frac{\sqrt{3}}{2}$ and the degree to which the alternative A_i dissatisfies the criterion C_j as $\frac{1}{2}$. It is easily seen that $\frac{\sqrt{3}}{2}+\frac{1}{2}\geq 1$. Thus, this situation cannot be described by using A's IFS. But we can employ the PFS to capture it because of $(\frac{\sqrt{3}}{2})^2+(\frac{1}{2})^2\leq 1$. That is to say, the PFSs involve more uncertainties than A's IFSs and are usually able to accommodate higher degrees of uncertainty. Accordingly, PFSs are more capable than A's IFSs of modeling the imprecise and imperfect information in practical decision making [48].

Ever since PFSs' appearance, many studies [6–7,31,33,43–44,48] have been conducted on multicriteria decision making (MCDM) methods with Pythagorean fuzzy information. For instance, Yager [43] developed an useful decision method based on Pythagorean fuzzy aggregation operators to handle Pythagorean fuzzy MCDM problems. Zhang and Xu [48] provided the detailed mathematical expression for PFS and introduced the concept of Pythagorean fuzzy number (PFN). Meanwhile, they also developed a Pythagorean fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) for handling the MCDM problem within PFNs. Yager and Abbasov [44] showed that the PFNs are a subclass of complex numbers called $\pi - i$ numbers and they also proposed a decision method to handle the MCDM problem in which the criteria values are expressed by $\pi - i$ numbers. Peng and Yang [31] proposed the division and subtraction operations for PFNs, and also developed a Pythagorean fuzzy superiority and inferiority ranking method to solve multicriteria group decision making problem with PFNs. Afterwards, Beliakov and James [6] focused on how the notion of "averaging" should be treated in the case of PFNs and how to ensure that the averaging aggregation functions produce outputs consistent with the case of ordinary fuzzy numbers. Reformat and Yager [33] applied the PFNs in handling the collaborative-based recommender system. In [7], Bustince et al. reviewed the definitions and basic properties of the different types of fuzzy sets that have appeared up to now in the literature, and pointed out that in any case A's IFSs are PFSs.

In human cognitive and decision-making activities, it is not completely justifiable or technically sound to quantify the degrees of the membership and non-membership in terms of a single numeric value [11,29]. Instead, it is convenient for the decision maker to employ intervals for expressing his/her preference about the membership function and the non-membership function. For this purpose, we further extend the PFS to propose the concept of interval-valued PFS (IVPFS) which is parallel to interval-valued intuitionistic fuzzy set (IVIFS) [4]. To our knowledge, IVIFSs have been successfully applied in various fields [10–13,15,21–22,34,38,40–42,45–46,50]. As an extension of IVIFSs from the perspective of the constraint condition, IVPFSs also have wide application fields, especially in MCDM fields such as the selection of suppliers in supplier chain management, evaluation of goods, etc. The elements in IVPFS are called interval-valued Pythagorean fuzzy numbers (IVPFNs). In order to further handle the MCDM problems under Pythagorean fuzzy environment based on PFNs and IVPFNs, it is necessary to develop correspondingly effective decision making methods.

First introduced by Paelinck [26–28], QUALIFLEX (qualitative flexible multiple criteria method), is one of the well-known outranking methods to solve the MCDM problems with crisp (non-fuzzy) numbers. This QUALIFLEX method is based on the pairwise comparisons of alternatives with respect to each criterion under all possible permutations of alternatives and identifies the optimal permutation with the maximum value of concordance/discordance index [24]. The most characteristic of the QUALIFLEX method is the correct treatment of cardinal and ordinal information [32]. The QUALIFLEX method can perfectly address the complex MCDM problems where a lot of criteria are utilized to assess a limited number of alternatives. This kind of problems can be easily found in many real-world situations, such as the medical decision-making problems concerning acute inflammatory demyelinating disease [14], natural resources management problems and the risk evaluation problems of strategic emerging industries as shown in Section 5.1, etc. Owing to the increasing complexity of decision making environment, the fuzzy sets or the generalizations of fuzzy sets are usually used by the decision maker to express his/her imprecise and uncertain preference information [8–9]. For this reason, several papers have recently been devoted to fuzzy extensions of the QUALIFLEX approach in the literature. For example, Chen et al. [14,39] proposed an extended QUALIFLEX method for handling the MCDM problems with interval type-2 trapezoidal fuzzy numbers (IT2TrFNs). Zhang and Xu [49] developed a hesitant fuzzy QUALIFLEX approach for dealing with MCDM problems in which both the criteria values and the weights of criteria are represented by hesitant fuzzy elements (HFEs). Chen [11] developed an interval-valued intuitionistic fuzzy QUALIFLEX method to address MCDM problems with IVIFSs.

Although the aforementioned QUALIFLEX method and its extensions are very effective for solving different real-world MCDM problems, they suffer from some limitations: (1) in several practical MCDM problems various types of relationships may exist among criteria [16,17,37], while these methods under the hypothesis that all criteria are independent fail to deal with such MCDM problems with criteria interactions; (2) the hierarchical structure which is very suitable for human thought process is usually used to decomposition the complex MCDM problem [5,30], but these methods cannot address such MCDM problems with hierarchical structure; (3) these existing methods also fail to deal synchronously with the PFN and IVPFN decision data in the decision process.