Information Sciences 330 (2016) 157–174

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Almost periodicity of set-valued functions and set dynamic equations on time scales☆

Shihuang Hong*, Yingzi Peng

Science College, Hangzhou Dianzi University, Hangzhou 310018, People's Republic of China

ARTICLE INFO

Article history:
Received 12 June 2013
Revised 4 July 2015
Accepted 6 October 2015
Available online 22 October 2015

Keywords:
Time scale
Almost periodicity
Set-valued function
Set dynamic equation

ABSTRACT

In this paper, we first propose the concept of the almost periodicity of set-valued functions on almost periodic time scales. Then based on the exponential dichotomy, we discuss the existence and uniqueness of almost periodic solutions of set dynamic equations on time scales. As an application, several examples are presented to exhibit that one can obtain the existence of almost periodic solutions to interval-valued dynamic equations (or fuzzy dynamic equations) and their discrete counterparts in a united means.

© 2015 Published by Elsevier Inc.

1. Introduction

The study of the periodicity of solutions is one of the most interesting and important topics in the qualitative theory of differential equations and difference equations due both to its mathematical interest and its applications in different areas, such as in engineering, life sciences, information sciences and control theory, among others. However, the conditions to guarantee the periodicity are very demanding. For this reason, in the past decades many authors have studied several extensions of the concept of periodicity and have established the existence of solutions with any such property for differential equations and their difference counterparts. We mention almost periodicity and asymptotically almost periodicity etc. Careful investigation reveals that it is similar to explore some qualitative problems of almost periodic differential equations and their discrete analogy in the approaches (see, e.g. [7,20,30,33,34,42]). It is natural to propose the concept of almost periodic time scales and almost periodic functions on them in [26–28] to unify the study of the above mentioned continuous and discrete problems and offer more general conclusions.

Hukuhara derivative (*H*-derivative, for short) of set-valued functions is the starting point for the topic of set differential equations (SDEs) [14] and later also for fuzzy differential equations (FDEs) [37]. In [22–24] the authors were concerned with the interrelation between SDEs and FDEs. SDEs (FDEs) and set(fuzzy) difference equations saw a rapid development over the last few years because of the advantages that this theory provides (e.g., differential systems [12,13,31,32,35,40,41], difference problems [11,24,25,39]). *H*-derivative on time scales has been introduced by Hong [16] and corresponding dynamic equations have been studied in [18,19,29] which are essentially important to get some insight into and better understanding of the subtle difference between SDEs (or FDEs) and their discrete analogous. However, this approach has the disadvantage that it leads to solutions

☆ This work was supported by Natural Science Foundation of China (grant no. 71471051) and Natural Science Foundation of Zhejiang Province (grant no. LY12A01002).

* Corresponding author. Tel.: +86 0571 86919035.
E-mail address: hongshh@hotmail.com (S. Hong).

http://dx.doi.org/10.1016/j.ins.2015.10.008
0020-0255/© 2015 Published by Elsevier Inc.

which have an increasing length of their support [10]. This results in a considerable impossibility of reaching the periodicity of solutions of SDEs (FDEs). The demand of an almost periodic solution such as a periodic solution is entirely different. Therefore, in [2] the author developed a theory of almost periodic fuzzy functions and discussed corresponding properties.

H-differentiability concept has another drawback exactly as pointed out in [38], namely, *H*-derivative depends on the existence of Hukuhara difference but the latter does not always exist. Recently, several generalized *H*-derivatives are proposed to overcome some shortcomings of this approach. For instance, the strongly generalized differentiability (*G*-differentiability) [1] was defined by considering lateral *H*-derivatives (four cases), the concept of *gH*-differentiability was introduced by the authors in [38], which is based on a generalization of the Hukuhara difference between two intervals and Chalco-Cano et al. in [6] defined *gH*- differentiability of set-valued functions from the real axis $\mathbb{R}$ into K_c^n and studied its relationship to *G*-differentiability.

In this paper we shall adopt *gH*-derivative of set-valued functions on time scales which is an extension of Δ_H-derivative introduced in [16]. Our purpose in this sequel is twofold. On the one hand, we shall present the concept of the almost periodicity of set-valued functions on almost periodic time scales and explore their main properties. We extend the notion so that one can explore some corresponding problems of SDEs and set difference equations, also, their fuzzy counterparts, in a unified way. On the other hand, we explore the existence of almost periodic solutions to SDEs on time scales and their applications. To this end, we first introduce the exponential dichotomy to linear set dynamic equations on time scales. Moreover, using this and a weakly contractive fixed point theorem we obtain the existence of almost periodic solutions to linear and nonlinear set dynamic equations on time scales, respectively. As an application of our results, we also discuss the existence and uniqueness of almost periodic solutions to some interval dynamic equations and fuzzy dynamic equations via some examples.

2. Preliminaries

In this section, we recall briefly the necessary background material for a self-contained presentation of our study. We first recall the notion of the time scale built by Hilger and Bohner. For more details, we refer the reader to [4,15].

A closed nonempty subset $\mathbb{T}$ of real axis $\mathbb{R}$ is called a time scale or measure chain. For $t \in \mathbb{T}$ we define the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ by $\sigma(t) = \inf\{\tau \in \mathbb{T} : \tau > t\}$, while the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ is defined by $\rho(t) = \sup\{\tau \in \mathbb{T} : \tau < t\}$. The function $\mu : \mathbb{T} \to [0, \infty)$ called the graininess function is defined by $\mu(t) = \sigma(t) - t$ for $t \in \mathbb{T}$. In this definition we put $\inf \emptyset = \sup \mathbb{T}$ (i.e. $\sigma(t) = t$ if $\mathbb{T}$ has a maximum t) and $\sup \emptyset = \inf \mathbb{T}$ (i.e. $\rho(t) = t$ if $\mathbb{T}$ has a minimum t), where $\emptyset$ denotes the empty set. t is said to be right scattered if $\sigma(t) > t$ and t is said to be right dense (rd) if $\sigma(t) = t$. t is said to be left scattered if $\rho(t) < t$ and t is said to be left dense (ld) if $\rho(t) = t$. In this paper we stipulate that the time scale $\mathbb{T}$ is $\mathbb{T} - \{M\}$ if $\mathbb{T}$ has a left scattered maximum M.

We continue with a description of the basic known results for Hausdorff metrics, continuity and differentiability for set-valued mappings on time scales and their corresponding properties within the framework of time scales. We refer readers to [16,24] for details. Let $\mathbb{R}^n$ denote the n-dimension real victor space and K_c^n denote the collection of nonempty compact and convex subsets of $\mathbb{R}^n$. The following operations can be naturally defined on it:

$$X + Y = \{x + y : x \in X, y \in Y\}, \quad \lambda X = \{\lambda x : x \in X\}, \ \lambda \in \mathbb{R}.$$
$$XY = \{xy : x \in X, y \in Y\} \quad \text{for } X, Y \in K_c^n.$$

Here, assume that the product operation in $\mathbb{R}^n$ is well defined. The set $Z \in K_c^n$ satisfying $X = Y + Z$ is known as the geometric difference (*H*-difference) of the set X and set Y and is denoted by the symbol $X -_H Y$. It is worthy to note that the geometric difference of two sets does not always exist but if it does it is unique. A generalization of geometric difference proposed in [38] aims to guarantee the existence of difference for any two intervals in K_c^1. In the light of this, a generalized difference called the g-difference, "$-_g$", can be defined for any $X, Y \in K_c^n$, that is

$$X -_g Y = Z \Leftrightarrow \begin{cases} (a) \quad X = Y + Z, \quad \text{or} \\ (b) \quad Y = X + (-1)Z. \end{cases}$$

It is clear if the g-difference exists, it is unique and it is a generalization of the geometric difference since $X -_g Y = X -_H Y$, whenever $X -_H Y$ exists. In addition, let $X, Y \in K_c^n$, the authors in [38] enumerated the following properties:

(i) $X -_g X = \{0\}$; $(X + Y) -_g Y = X$; $\{0\} -_g (X -_g Y) = (-Y) -_g (-X)$.
(ii) If $X -_g Y$ exists in the sense (a), then $Y -_g X$ exists in the sense (b) and vice versa.
(iii) $X -_g Y = Y -_g X = Z$ if and only if $Z = \{0\}$ and $X = Y$.

Throughout this paper, we always assume that the g-difference of any two elements under consideration in K_c^n exists. We remark that the assumption may be valid, for instance, in the unidimensional case (with $K_c^1 = \mathbb{I}$, a class of all closed bounded intervals of the real line) the g-difference exists for any two compact intervals.

We define the Hausdorff metric as

$$D[X, Y] = \max\left\{\sup_{y \in Y} d(y, X), \sup_{x \in X} d(x, Y)\right\},$$

where $d(x, Y) = \inf\{d(x, y) : y \in Y\}$ and X, Y are bounded subsets of $\mathbb{R}^n$. It is obvious that the Hausdorff metric satisfies relations of the ordinary metric.

Download English Version:

https://daneshyari.com/en/article/392845

Download Persian Version:

https://daneshyari.com/article/392845

Daneshyari.com