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Heuristic aggregation of individual judgments in AHP group decision making using simulated annealing algorithm

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ABSTRACT

There are various aggregation procedures for obtaining a group priority vector within Analytic Hierarchy Process (AHP) supported decision making processes. This paper will introduce a heuristic aggregation procedure based on simulated annealing (SA) algorithm to be used for the purposes of obtaining a group priority vector at any node of an AHP hierarchy. The proposed procedure performs its aggregation process by minimizing the group Euclidean distance (*GED*) (consensus measure) across group weights and judgments, and the group vector obtained in this way is invariant to any prioritization method. In other words, there is no need to have individual priority vectors as is required by some other aggregation procedures. Along with SA minimization of the *GED*, the group rank reversal (minimum violation) criterion is implemented as a control mechanism, as well as the consensus measure based on the ranks of alternatives. The presented procedure is compared with several reported combinations of different prioritization methods and group aggregation procedures. Five examples from literature are used to show that the proposed procedure performs better or at least equally to several other well known combinations of prioritization and aggregation in AHP group decision making frameworks.

1. Introduction

The Analytic Hierarchy Process (AHP) [28] is a multi criteria decision-making method that has been used in many applications related to decision-making problems [17], and is applicable to both individual and group decision making situations. The two most used procedures to obtain group priorities in the AHP are the aggregation of individual judgments (AIJ) and the aggregation of individual priorities (AIP) [1,13,26]. In situations which require the AIJ, the group matrix can be generated using consensus, voting or the mathematical aggregation of individual judgments, after which the group's priorities can be calculated using any prioritization method. However, because in many group settings it is difficult to achieve consensus and obtain an accurate and successful round of voting, the usual choice for generating the group matrix ends up being mathematical aggregation. Geometric mean aggregation is suggested for the AIJ in order to preserve the reciprocal property of the generated group matrix and to account for possible significant differences in individual decision makers' judgments [13]. In the second case (AIP), the priorities of decision makers (DMs) are first calculated using the selected prioritization method, after which individual priorities are aggregated using either the weighted geometric mean or weighted arithmetic mean method, though Ishizaka and Labib [18] encourage using the weighted arithmetic mean method.

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There is also a third procedure that should be considered for use in the context of group aggregation: the consensus convergence model (CCM) [21], where by repeatable mathematical procedure and through mutual respect the decision makers not only achieve consensus on the issue under consideration but also agree on the overall relative weight of each decision maker [32]. The original model assumes that DMs have opinions about the expertise and rationality of other DMs. The model proposed by Regan et al. [27] uses a weight of respect based on the strength of the difference in the alternative weights assigned by all DMs, and when used in combination with the AHP, priority vectors obtained from the individual judgment matrices of all DMs represent the input data to this model [32].

Group priorities can also be obtained by combining the AHP with compatible 'soft' consensus models [e.g., 7,11,25,35–37]. Consensus is usually defined as the unanimous agreement of involved DMs after possible alternatives have been considered, but because more often than not unanimous consensus is impossible to achieve in real life [24], success of these procedures depends on correctly measuring the consensus degree, i.e. to measure the consistency of individual decisions against the group decision. The main idea is to generate advice on how the DMs with the highest inconsistencies with the group decision should change their preferences and contribute to the group consistency. In this study, the geometric cardinal consensus model (GCCM) [11], one of the most cited models, is used. This model requires the use of the logarithmic least squares (LLS) prioritization method and the geometric cardinal consensus index (*GCCI*) to measure consensus; an index is based on the geometric consistency index (*GCI*) given by Crawford and Williams [9]. The GCCM automatically changes individual judgments of DMs with the highest *GCCI* values according to the group priority vector, and this model has two desired features: firstly, in reaching a consensus the adjusted judgment matrix has a better individual consistency index (i.e., *GCI*) than the corresponding original judgment matrix, and secondly it is preserved that the Pareto principle of social choice theory is satisfied. We also find that this model is correct in the aforementioned sense although we are aware that the problem with the implementation of the model could be that it automatically modifies DMs' opinions to reach consensus while in real life situations sometimes the DM may not want to change his/her opinion even if he/she demonstrates the highest inconsistency with the group decision.

Determining the effectiveness of the AIJ, AIP or CCM as group aggregation procedures requires acknowledging that group priorities will also depend on the selected prioritization method. This is because in AHP-supported group decision processes it is quite often impossible to expect full consistency from both the individual and group pairwise comparison matrices, especially if the order of matrices is higher than three. Srdjevic [30] compared six prioritization methods against two consistency measures (Euclidean distance and minimum violation), and concluded that, though being quite simple, the additive normalization method (AN) is highly competitive with other more sophisticated methods such as, e.g., eigenvector method (EV) [28] or logarithmic least square method (LLS) [9]. Athough comparison analyses of prioritization methods are carried out by many authors [6,9,10,14,20,22], so far there is no clear agreement on which method is the best; rather, an open discussion is maintained on the effectiveness of different prioritization methods. In our study we used prioritization methods EV, AN and LLS. It is important to note here that when using EV and AN, the group priority vector obtained by the AIP and AIJ can be different, while in case of LLS, the AIJ and AIP produce the same group priority vector [3].

In this paper we propose a heuristic stochastic approach to group decision making based on the use of the simulated annealing (SA) algorithm, an aggregation procedure which searches for the best group priority vector for a given node in an AHP hierarchy. The precision of the group priority vector is then further refined by minimizing the group Euclidean distance (*GED*) of all individual judgments from group priority vectors, which are collected during the iterative SA run. Following the logic that the commitment of the group to the implementation of the outcomes depends upon the level of consensus achieved by the group [24], we believe that minimization of *GED* can be considered as an objective search for maximum consensus between individuals within the group. The group priority vector obtained in this way is invariant to any prioritization method; that is, there is no need to have individual priority vectors as is required by some other aggregation procedures.

In order to check the validity of our approach, five examples are used to compare our results with results obtained by various combinations of aggregations (AIJ and AIP), consensus models (CCM and GCCM) and prioritization methods (EV, AN and LLS), and hereafter these combinations will be referred to as 'aggregation schemes'. For comparison purposes, all results obtained by our and other aggregation schemes are 'controlled' by two well known criteria, group minimum violation (*GMV*) measure, which we also purposely introduce in our approach, and the consensus measure (*CM*) proposed by Herrera-Viedma et al. [16].

The structure of the paper is as follows. In Section 2 we present preliminary knowledge about the most commonly used prioritization methods and group aggregation procedures within the AHP. Section 3 introduces the proposed simulated annealing aggregation procedure (SAAP). Section 4 describes criteria used for comparison of the results obtained by SAAP and other aggregation schemes. In Section 5, a Wilcoxon matched-pairs signed-ranks nonparametric test is described as it was carried out to ascertain whether the application of SAAP and other aggregation schemes produce significant differences in the values of tested criteria. Five numerical examples are given in Section 6 followed by discussion in Section 7 on the philosophical implications and benefits of applying SAAP compared to the other tested consensus models. Section 8 closes the paper with concluding remarks.

2. Preliminary knowledge

2.1. The problem statement

The group prioritization problem of n elements E_1, E_2, … , E_n at a given level of the AHP hierarchy can be formalized in the following way. The decision maker k semantically compares any two elements, E_i and E_j, and indirectly (verbally) or directly (numerically) assigns a value $a_{ij}^{(k)}$ which represents his judgment of the relative importance of decision element E_i over E_j