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An axiomatizable logical foundation for lattice-ordered qualitative linguistic approach for reasoning with words

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ABSTRACT

It is commonsense that people do express, think, reason, and make decision directly using linguistic terms in natural language rather than using numerical quantification. How to establish the formalized approach imitating the common way of human being's to manipulate directly linguistic terms without numerical approximation to provide the rational decision is still an open research area. Investigations on the algebraic structure of linguistic term set for varied decision making applications (especially in social science) still lack a formalism for development of strict linguistic valued logic system as a theoretical foundation and its approximate reasoning scheme in practice. To attain this goal we characterize and construct a typical structure of commonly used linguistic term sets in natural language by a lattice-ordered algebra structure – lattice implication algebra (LIA), where Łukasiewicz implication algebra, as a special case of LIA, plays a substantial role. By using Łukasiewicz logic's axiomatizability in terms of Pavelka type fuzzy logic, we propose a new axiomatizable lattice ordered qualitative linguistic truth-valued logic system based on LIA to place an important foundation for further establishing formal linguistic truth-valued logic based approximate reasoning and decision making with applications. This proposed logic system has a distinct feature of handling comparable or incomparable linguistic terms directly without numerical quantification, will be especially beneficial for perception-based decision making processes. It attempts to enhance the quantitative theory of decision science with qualitative, algebraic and logic-oriented approaches to achieve reasoning with words.
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1. Introduction

There is much qualitative information in the area of evaluation processes and decision making in real life, like human beings' subjective judgments. Human beings cannot be seen as a precision mechanism, they usually express their knowledge or subjective judgments about the world using natural language which is full of vague and imprecise concepts. A linguistic term differs from a numerical one in that its values are not numbers, but words or sentences in a natural or artificial language. The use of linguistic terms implies processes of Computing with Words (CWW) [93]. Among various computational approaches in the literature, CWW on the basis of fuzzy set theory, where membership functions (as a numerical quantification of linguistic terms) are generally at the core, has been developed intensively since 1965 [90] as well as applied in various areas, particularly in the area of artificial intelligence.

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In order to set up some ways to deal with human perception-based reasoning and decision making problem, there exist alternative methods developed recently to model and compute with linguistic information in natural languages from the different points of view, so called *linguistic-valued based intelligent information process approaches* [68]. These approaches tried to modify and overcome some limitations of fuzzy-set theories based CWW, e.g., difficulty in determining and interpreting fuzzy set membership functions of linguistic values, computational complexity and loss of information due to linguistic approximations. A key insight of linguistic-valued based approaches reflects the use of "words" as computational values, i.e., the symbolic approach acts by the direct computation, manipulation and reasoning on the available linguistic terms in natural languages from the ordinal, the algebraic or the symbolic logic point of view [68]. This assumption is reasonable and realistic in some real world applications because it might be more natural and easier for the decision maker to express qualitative linguistic evaluation among decision alternatives than to provide a quantitative measure of degree. And it is commonsense that people do express, think, reason, and make decision directly using linguistic terms in natural language rather than using numerical quantification [36]. Therefore, the application of this kind of approaches is beneficial because it introduces a more flexible framework for representing the information in a more direct and suitable way when it is not possible to express it accurately. Thus, the burden of quantifying a qualitative concept is eliminated and the systems can be simplified. The linguistic-valued based intelligent information process approach aims to offer a natural qualitative framework for handling imprecise information in decision making.

Although there have been some investigations of the linguistic-valued based intelligent information process approach together with some applications in decision making and social science [2,52,68] and references therein, one of the key problems has not been paid sufficient attention, let alone solved. That is, the theoretical foundation of those linguistic-valued based intelligent information process approaches, e.g., how to model linguistic-values set with appropriate algebraic structure? How to establish appropriate formalism for the development of logic systems based on linguistic truth values algebra, as well as approximate reasoning and automated reasoning based on linguistic truth-valued logic systems? Some key issues need to be considered accordingly by exploring the feature of the linguistic terms used in natural language.

First of all, considering the linguistic terms used in natural language, it is easy to observe that one can compare meanings of some linguistic terms and discover an ordering relationship on a linguistic domain based on intuitive meaning of linguistic terms. For example, it is clear that *true* > *false*, *very true* > *more* true and so on. However, the method embedding mapping from the linguistic value domain into fuzzy set does not preserve the discovered ordering relationship, which is another limitation of fuzzy set based approaches as stated in [36]. On the other hand, another important feature of linguistic terms in natural language is that there are also some "vague overlap areas" among some linguistic terms, which cannot be strictly linearly ordered, e.g., *highly true* and *slightly false* are incomparable; and *approximately true*, *possibly true*, and *more or less true* are also incomparable. One cannot collapse that structure into a linearly ordered structure, because then one would impose an ordering on them, which was originally not present. This means the set of linguistic values may not be strictly linearly ordered. Accordingly, how to handle the incomparable linguistic information in linguistic-valued based information processing remains a great challenge, which has not yet been paid sufficient attention so far.

Furthermore, from the symbolism point of view, it is important and necessary to establish the logical foundation for decision making approach. Logic serves as the foundation and standard for justifying or evaluating the soundness and consistency of the methods, including decision making methods [6]. In order to establish the rational reasoning approaches and intelligent decision support system to deal with both totally ordered information and non-totally ordered information, it is important and necessary to study the corresponding logical foundation, which should be some kind of non-classical logical systems. As it is well-known that a partially ordered structure or a lattice structure (with much richer properties than partially ordered structures) has been a suitable tool to order objects, and has significant applications in other areas of mathematics, computer science, and physics. Especially, the lattice structure has been a very useful and well-developed branch of abstract algebra for modeling the ordering relations in real-world problems, and the general framework of lattice theory is almost indispensable in explaining complex phenomena in an easy way [4,17,21,55]. Therefore, lattice-valued logic [82] is thus an important and promising research direction which can serve as a logical foundation for ordering based decision making by taking a general lattice as its truth-valued field, which empowers it the ability to model and manipulate fuzziness and incomparability occurring in people's thinking and decision-making in a logical way.

Lattice-valued logics have been investigated initially by Goguen [18,19] who extended the concept of fuzzy sets to that of L-fuzzy sets in which membership grades form a partially ordered set instead of a linearly ordered set such as [0,1]. He also proposed the first lattice-valued logic formal system based on complete-lattice-ordered semigroups, which is also called a complete residuated lattice in important lattice-value logics investigated in [56,62], called Pavelka-type lattice-valued logic. Afterwards, many researchers devoted themselves to formal deduction, the correlation between semantics and syntax within the related many-valued logic and lattice-valued systems, such as [5,11,20,25,57–60], and among others. However, there is still a gap between these algebraic, logic and reasoning formalisms and solving decision making in the real practice.

Based on the above academic ideas, this present work focuses on lattice-ordered symbolic linguistic-valued based approaches and aims at establishing an axiomatizable (i.e., a sound and complete) linguistic valued logic system, which is based on constructing a comparatively appropriate linguistic truth-valued algebraic structure by using a lattice-ordered algebra structure, providing a comparatively appropriate interpretation for the logical formulae. The work has been briefly outlined in [48]. This paper aims at extending, refining, completing and systematizing the results in [48]. This work will place

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