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Estimation of distribution algorithm for a class of nonlinear bilevel programming problems

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ABSTRACT

In this paper, a novel evolutionary algorithm called estimation of distribution algorithm (EDA) is proposed for solving a special class of nonlinear bilevel programming problems (BLPPs) in which the lower level problem is a convex programming problem for each given upper level decision. This special type of BLPP is transformed into a equivalent single-level constrained optimization problem using the Karush-Kuhn-er conditions of the lower level problem. Then, we propose an EDA based on the statistical information of the superior candidate solutions to solve the transformed problem. We stress that the new population of individuals is sampled from the probabilistic distribution of those superior solutions. Thus, one of the main advantages of EDA over most other meta-heuristics is its ability to adapt the operators to the structure of the problem, although adaptation in EDA is usually limited by the initial choice of the probabilistic model. In addition, two specific rules are established in the initialization procedure to make use of the hierarchical structure of BLPPs and to handle the constraints. Moreover, without requiring the differentiability of the objective function, or the convexity of the search space of the equivalent problem, the proposed algorithm can address nonlinear BLPPs with non-differentiable or non-convex upper level objective function and upper level constraint functions. Finally, the proposed algorithm has been applied to 16 benchmark problem; in five of these problems, all of the upper level variables and lower level variables are 10-dimensional. The numerical results compared with those of other methods reveal the feasibility and effectiveness of the proposed algorithm.

1. Introduction

The bilevel programming problem (BLPP) addresses an optimization problem whose constraint region is determined implicitly by another (parametric) mathematical programming problem [15]. This nested system arises when two decision makers, ordered within a hierarchical structure, may have conflicting objectives. Considering the possible reactions of the decision maker at the lower level (the follower), the decision maker at the upper level (the leader) optimizes his/her objective function first. Afterward, the follower selects his/her decision under the given decision of the leader. Because plentiful real-life problems, such as transportation, management and economics, engineering design, supply chain planning, principal-agent problems, and health insurance, can be transformed into BLPPs [11–13,19,38,52,54], it is significant to solve these problems in an efficient way [7,14,29,39,44,53].

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However, the bilevel programming problem is non-convex and non-differentiable. Even the linear bilevel programming problem, the simplest version of BLPPs, was shown to be NP-hard by Jeroslow [21]. This result was later proved by Bard [6] and Ben-Ayed and Blair [8]. Afterward, Vicente et al. [40] confirmed that even searching for the local optima of the linear BLPPs is NP-hard. See Ref. [17] for more detailed discussion of the complexity issues in the linear bilevel programming.

Over the past thirty years, much progress has been made in developing solution methods for BLPPs. Such methods can generally be classified into the following categories [14,44]: extreme-point search approaches for the linear case, branch-and-bound approaches, complementary pivoting methods, decent methods, penalty function methods and trust-region methods. Because methods that are based on meta-heuristics do not require differentiability of the objective functions, gradient information, or convexity of the search space, they have attracted considerable attention for their potential as alternative methods for bilevel programming problems. Heuristic algorithms (including simulated annealing, neural network, particle swarm optimization, genetic algorithms, tabu search, bee colony algorithm, cuckoo optimization algorithm and chaotic annealing) and their applications have received a large amount of attention over the past few years [18,22,23,31,36,37,41,44–50]. This attention has motivated the use of heuristic algorithms for bilevel programming problems [44].

As a new meta-heuristic, an estimation of distribution algorithm (EDA) is one of the stochastic optimization techniques that explores the space of potential solutions by building and sampling explicit probabilistic models of promising candidate solutions. This model-based approach to optimization has allowed EDAs to solve many large and complex problems, such as multi-objective knapsack, military antenna design, identification of clusters of genes with similar expression profiles, economic dispatch, forest management, portfolio management, cancer chemotherapy optimization, and environmental monitoring network design (see [20] and the references therein). It is important to stress that no other techniques were shown to be capable of achieving better performance or solving problems of comparable size and complexity than EDAs in most of these applications. In this paper, we extend the application of EDA to address bilevel programming problems as an attempt. Notably, there are no reports on solving BLPPs through EDA-type methods. Through building an explicit probabilistic model for the selected candidate solutions that have been obtained so far, EDAs extract the global statistical information of the promising solutions. Then, an EDA method improves the quality of the candidate solutions by generating the new generation of solutions via sampling based on the model, instead of via the GA-type operators of crossover and mutation. Thereby, EDA surpasses other intelligent algorithms for efficiency because of its avoidance of the complex parameter selection [26]. Similar to other meta-heuristics methods, EDA is implemented without requiring the differentiability of the objective function or the convexity of the search space.

In this paper, we mainly concentrate on the nonlinear bilevel programming problem, in which both the upper level objective function and the lower level objective function are all real-valued convex functions. First, we transform the nonlinear bilevel programming problem into a single-level mathematical programming problem by replacing the lower level problem with its Karush–Kuhn–Tucker (KKT) conditions. To handle the original upper level constraints and the KKT conditions, a new fitness function with a penalty scheme is constructed to evaluate the quality of each candidate solution. In solving constrained optimization, addressing an infeasible solution is very difficult. Hence, we initially generate the first generation, while obeying two preset rules: (i) the upper level decision of each individual belongs to the projection of the feasible solution region onto the solution space of the upper level problem, (ii) the lower level decision of each candidate solves the lower level problem under the corresponding given upper level decision. Through the above rules, we can initially generate the solution in the inducible region of the BLPP to improve the efficiency of the algorithm. Then, in each generation, we select the superior solutions based on their fitness function values and build up a probabilistic model for the selected feasible solutions. Afterward, the algorithm generates new populations based on the model and regularizes new individuals to satisfy the ordinary restrictions. Note that the proposed EDA calls for no special characters of the upper level objective function and constraint functions. Thus, it can effectively address the nonlinear bilevel programming problem with a non-differentiable upper level objective function and non-differentiable or non-convex upper level constraint functions, only if the lower level problem is convex for each given upper level decision and the solution to the problem (BLPP) exists.

The organization of the remainder of this paper is as follows. The general formulation and basic concepts of BLPP are presented in Section 2. After a brief recall of the estimation of distribution algorithm, the proposed algorithm for the nonlinear BLPP is given after the reformulation of BLPP in Section 3. Experimental results regarding 16 benchmark problems are presented in Section 4 and the paper concludes with a summary in Section 5.

2. The bilevel programming problem

2.1. The general formulation and basic concepts for BLPP

The general formulation of a bilevel programming problem can be stated as follows:

$$(BLPP)\ \min_{x\in X, y\in Y} F(x,y) \tag{1}$$

$$s.t.\quad G(x,y) \leqslant 0, \tag{2}$$

where y, for each vector x, belongs to the solution set of the so-called lower level problem: