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Optimal topological design for distributed estimation over sensor networks

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ABSTRACT

The topological structure of sensor network possesses distinctive and interesting characteristics that are important for many applications. In the previous work by Liu et al. (Y. Liu, C. Li, W.K.S. Tang, Z. Zhang, Distributed estimation over complex networks, Inform. Sci. 197(8) (2012) 91–104) the effects of network topology on distributed estimation have been addressed. In this paper, we further focus on the optimal topological design of sensor networks, which targets for improving the performance of distributed estimation. Based on spectral analysis, it is shown that this design problem is equivalent to finding an optimal topology that maximizes the eigenratio of the second smallest and the largest eigenvalues of the respective network Laplacian matrix. To tackle this optimization problem, a computational algorithm combining a local greedy algorithm and tabu search is proposed, in which the constraint on the distance of two communicated sensors is incorporated. As shown in the numerical simulations, the proposed algorithm outperforms other optimization strategies in the viewpoints of accuracy, robustness and complexity. Consequently, the quality of distributed estimation can be improved by obtaining a better network topology.

1. Introduction

Sensor networks have received considerable research interest due to their wide applications in signal processing, industrial control, precision agriculture, environmental monitoring, military surveillance, robotics, and so on [1,4,6,7,21]. A sensor network usually consists of a large number of spatially densely distributed sensors equipped with microelectronic devices. The sensors share information and communicate with others to perform some tasks of interest. However, since sensors are usually battery operated, they have limited power resources, computational ability and communication capability. It is therefore inefficient, if not impossible, to transmit information over the entire sensor network. Instead, it is natural to complete the task by carrying out local data fusion based on the information collected from a set of neighbor sensors. Therefore, distributed processing technique has currently become a major tenet for sensor networks [9,28].

In this paper, we focus on distributed estimation, one of the most important applications of distributed processing [5,9,20,23–25,32]. It aims to provide high accuracy of the estimation for some parameters of interest, relying on data collected at distributed sensors. By fusing the local information in certain collaborative manner, an accurate estimation can be achieved, while the amount of data communication and processing can be greatly reduced as compared with the centralized scheme. Thus, distributed estimation can save bandwidth and energy, which in turn extends the network lifetime and reduces latency [9].

In order to perform distributed estimation in large-size sensor networks, diffusion schemes [5,20,23,25,32] are usually recommended. They eliminate the requirement of the cyclic path through the network as in the incremental scheme [24],

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so that each sensor simply exchanges information with a set of sensors within its neighborhood. Concomitantly, as investigated in [22], diffusion algorithms depend on the connectivity among sensors, i.e. the network topology.

Then, a natural question arises: Given fixed numbers of sensors and links (if two sensors exchange information with each other, we assume there is a link between them), how to design an optimal network topology that maximizes the estimation performance whilst has good robustness and low complexity? In this paper, using the spectral analysis, we find that the considered problem can be achieved by maximizing the ratio of the second smallest and the largest eigenvalue (also known as the eigenratio) of the network Laplacian matrix, which is consistent with the optimization of network synchronizability [26,29]. That is to say, those optimization techniques initially used for improving network synchronizability are also applicable to our considered optimization problem. Then, the optimization algorithm proposed in [33] is further extended to design optimal network topology for distributed estimation. To agree with the practical cases that sensors are usually power-limited, the distance between communicated sensors is confined within a certain range.

To the best of our knowledge, this is the first time to discuss the optimal topological design for in-network distributed estimation. This problem is essential in practice. Sensors commonly work in harsh noisy environments (low SNR), and thus may provide poor estimation. In such cases, a few dB deduction in estimation errors seems to be extremely significant. Besides, considering that sensor failure always exists, it is more desirable to design sensor networks with good robustness. We show that the optimized networks obtained from the proposed optimization strategy have larger eigenratios, better performance of distributed estimation as well as robustness. Therefore, this study may provide a guide for the design of practical network topology.

The organization of this paper is as follows. In Section 2, a distributed diffusion LMS algorithm is briefly revisited and a new modified Laplacian cooperation rule is proposed. In Section 3, the optimization of distributed estimation with regard to network topological design is formulated. Theoretical analyses in both mean and mean-square senses indicate that this problem is equivalent to designing an optimal network topology, which maximizes the eigenratio of the network Laplacian matrix. In Section 4, an optimization strategy is proposed to tackle this optimization problem. In Section 5, the performances of the proposed algorithm in optimizing network topology and furthermore in enhancing distributed estimation are verified by simulations. Finally, this work is concluded in Section 6.

2. Distributed estimation over sensor networks

Consider a set of N sensors forming a network deployed for estimating a vector of interest. The sensor network can be represented by an undirected graph, $\mathcal{G} = (V; E)$, where V and E are the sets of nodes (i.e. the sensors) and links, respectively, such that a link connecting two sensors implies the availability of direct communication channel between them.

In the problem of diffusion estimation [5,22,25,31], every node k can access to a scalar observation $d_k(i)$ and an M-dimensional row regression vector $u_{k,i}$, at each time instant i, corresponding to a time realization of a random process $\{\boldsymbol{d}_k(i), \boldsymbol{u}_{k,i}\}$. Commonly, the following linear model is assumed:

$$d_k(i) = u_{k,i}w^o + v_k(i), \tag{1}$$

where w^o is an M-dimensional vector of interest and $v_k(i)$ is the background noise.

Now, we consider the distributed least-mean-squares (LMS)-like algorithm, which is known as diffusion LMS [5,25,32]. It is recognized as a good candidate for distributed estimation since it not only requires less complexity for both communications and computations, but also inherits the robustness of LMS implementation [31].

Mathematically, the diffusion LMS can be implemented as follows [5,22,25]. Let ψ_k^i be the estimate of w^o by sensor k at time instant i. Start with the initial value $\psi_k^{-1} = 0$, for each time instant $i \geqslant 0$, the estimate is updated by

$$\phi_k^{i-1} = \sum_{l \in N_k} c_{l,k}\psi_l^{i-1}, \tag{2a}$$

$$\psi_k^i = \phi_k^{i-1} + \mu_k u_{k,i}^*(d_k(i) - u_{k,i}\phi_k^{i-1}), \tag{2b}$$

where N_k denotes the set of sensors that directly connect to sensor k, $\mu_k > 0$ is the step-size at sensor k, and the real non-negative constants $c_{l,k}$ satisfy the following conditions:

$$\sum_{l \in N_k} c_{l,k} = 1, \quad \text{and } c_{l,k} = 0 \quad \text{for all } l \notin N_k. \tag{3}$$

As reflected in (2a), the past estimation ψ_l^{i-1} from the neighborhood of sensor k, are collected to form ϕ_k^{i-1}, and then the sensor k adaptively updates its own data $\{d_k(i), u_{k,i}\}$ to obtain a new estimate ψ_k^i as in (2b).

The coefficients $c_{l,k}$ represent the cooperative rule among sensors, which is determined by the network topology. In literature, several collaborative protocols have been suggested, such as Metropolis rule [5,25,32], Laplacian matrix [5], and adaptive combiners [32]. However, the relationship between $c_{l,k}$ and the network topology is inexplicit, making it difficult to optimize the estimation with respect to topological design.

Motivated by the study in distributed consensus [19], we propose a new cooperative rule related to the eigenvalues of the Laplacian matrix of the sensor network $\mathcal{G}$ as follows:

$$C = I_N - \alpha\mathcal{L}, \tag{4}$$