Information Sciences 328 (2016) 15–25

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Posynomial geometric programming problem subject to max–min fuzzy relation equations

Xue-Gang Zhou[a,b], Xiao-Peng Yang[a,c], Bing-Yuan Cao[a,d,*]

[a] School of Mathematics and Information Science, Key Laboratory of Mathematics and Interdisciplinary Sciences of Guangdong, Higher Education Institutes, Guangzhou University, Guangzhou, Guangdong, 510006, China
[b] Department of Applied Mathematics, Guangdong University of Finance, Guangzhou, Guangdong, 510521, China
[c] Department of Mathematics and Statistics, Hanshan Normal University, Chaozhou, Guangdong, China
[d] Guangzhou Vocational College of Science and Technology, Guangzhou, Guangdong, 510550, China

ARTICLE INFO

Article history:
Received 22 July 2014
Revised 16 July 2015
Accepted 29 July 2015
Available online 24 August 2015

Keywords:
Posynomial geometric programming
Fuzzy relation equation
Max–min composition

ABSTRACT

We discuss a class of posynomial geometric programming problem(PGPF), aimed at minimizing a posynomial subject to fuzzy relational equations with max–min composition. By introducing auxiliary variables, we convert the PGPF into an equivalent programming problem whose objective function is a non-decreasing function with an auxiliary variable. We show that an optimal solution consists of a maximum feasible solution and one of the minimal feasible solutions by an equivalent programming problem. In addition, we introduce some rules for simplifying the problem. Then by using a branch and bound method and fuzzy relational equations (FRE) path, we present an algorithm to obtain an optimal solution to the PGPF. Finally, numerical examples are provided to illustrate the steps of the procedure.

1. Introduction

In this study, we consider a certain type of posynomial geometric programming problems subject to max–min fuzzy relation equations described as follows:

$$\begin{aligned}&(\text{PGPF})\min z(x)=\sum_{k=1}^{p}c_k\prod_{j=1}^{n}x_j^{\gamma_{k_j}}\\&s.t. A\circ x=b,\end{aligned}\tag{1}$$

where $A=(a_{ij})_{m\times n}$, $x=(x_j)_{n\times 1}$, $b=(b_i)_{m\times 1}$, $a_{ij}, x_j, b_i\in[0,1]$, $c_k, \gamma_{k_j}\in R$, $c_k>0$, $i\in I=\{1,2,\dots,m\}$, $j\in J=\{1,2,\dots,n\}$, $k\in K=\{1,2,\dots,p\}$, and for given $j\in J$, $\gamma_{k_j}(k\in K)$ are either all non-positive real numbers or all non-negative real numbers. Without loss of generality, we assume that problem (1) satisfies the following inequalities:

$$1\geqslant b_1\geqslant b_2\geqslant\cdots\geqslant b_m\geqslant 0.$$

Otherwise, rearrange the components of b in decreasing order and adjust the rows of A accordingly b.

Fuzzy relational equations have played an important role in fuzzy set theory and fuzzy logic systems, and many researchers have discussed fuzzy relation equations based on different compositions of fuzzy relations, including max–min fuzzy relational

* Corresponding author. Tel.: +8615876504405.
E-mail address: caobingy@163.com, happyyangxp@163.com (B.-Y. Cao).

http://dx.doi.org/10.1016/j.ins.2015.07.058
0020-0255/

equations [29,30], max-product fuzzy relational equations [1], continuous t-norm fuzzy relational equations [5], interval-valued fuzzy relation equations [16], and so on. It is worth noting that the complete solution set of continuous t-norm fuzzy relational equations can be completely determined by a unique maximum solution and a finite number of minimal solutions [4,5,14,16]. Computing the maximum solution is easy, but finding all the potential minimal solutions of fuzzy relational equations is an NP-hard problem [3,25]. Recently, studies on fuzzy relational equations have been extended to the setting of interval-valued equations with max-t-norm composition. Wang et al. [35] have proposed three types of solution sets for this type of equation. Li and Fang [18] presented the detailed analysis of the classification and the solvability of general fuzzy relational equations with various compositions. Applications of fuzzy relational equations can be found in [5,12,26,27].

Wang et al. [34] first studied a class of latticized linear programming subject to max–min fuzzy relation inequalities. Li and Fang [19] considered the latticized linear optimization problem and its variants, which are a special class of optimization problems constrained by fuzzy relational equations or inequalities. Li and Wang [17] investigated the latticized linear programming that is subject to the fuzzy-relation inequality constraints with the max–min composition by using a semi-tensor product method, and proposed a matrix approach to this problem. Fang and Li [6] discussed an optimization model with a linear objective function subject to max–min fuzzy relational equations. They converted such an optimization model into a 0–1 integer programming problem and solved it by the branch-and-bound method. Wu and Guu [38] proposed a necessary condition for an optimal solution to explore the same optimization problem with positive cost coefficients in the objective function. Loetamonphong and Fang [22] considered a minimization problem with a linear objective function and a max-product fuzzy relational equations constraint. By using the nonnegative and negative coefficients in the objective function, this optimization problem is separated into two subproblems. The maximum solution of max-product fuzzy relational equations is an optimal solution to a subproblem formed by negative coefficients. In addition, the subproblem formed by the nonnegative coefficients can be converted into a 0–1 integer programming problem which can be solved by the branch-and-bound method. Guu and Wu [13] provided a necessary condition for an optimal solution in terms of the maximum solution derived from the fuzzy relational equations. We employed this necessary condition to provide an efficient procedure for solving the minimization problem. Wu and Guu [37] extended the fuzzy relational constraints with a max-product composition to the situation with max-strict t-norm composition. They proved that the necessary condition for an optimal solution in terms of the maximum solution also can be applied to the situation of max-strict t-norm composition. Peeva [28] found all the minimal solutions and compared their corresponding objective function values in order to obtain optimal solutions. Wang [33] investigated multiple linear objective functions subject to max–min fuzzy relational equations. Lu et al. [24] proposed a genetic algorithm for the problems where the objective function is a single nonlinear objective function and the constraints are max–min fuzzy relational inequalities. Loetamonphong et al. [23] presented the nonlinear multi-objective optimization problem with a fuzzy relational equation constraint. Ghodousian and Khorram [8] focused on subset of these problems where the solutions are fuzzy relational equations with max-prod composition and the objective function is linear. Khorram and Ghodousian [15] presented an optimization model with a linear objective function subject to a system of the fuzzy relational equations with max-av composition. Ghodousian and Khorram [9] investigated linear programming problem with the convex combination of the max–min and the max-average fuzzy relational equations. Li and Fang [20] considered the problem of minimizing a linear fractional function subject to a system of sup-T equations, where T means a continuous Archimedean triangular norm. Li and Fang [21] considered the detailed analysis of the resolution and optimization of a system of sup-T equations. Shieh [31] examined the feasibility of minimizing a linear objective function subject to a max-t fuzzy relation equation constraint, where t is a continuous/Archimedean t-norm. Ghodousian and Khorram [10] discussed linear optimization with an arbitrary fuzzy relational inequality. They proved that its optimal solution can be obtained if the problem is defined by a non-decreasing or non-increasing function. Freson et al. [7] considered a generalization of the linear optimization problem with fuzzy relational (in)equality constraints by allowing for bipolar max–min constraints, i.e. constraints in which not only the independent variables but also their negations occur. Guo et al. [11], Chang and Shieh [2] also proposed some rules to reduce linear optimization subject to the fuzzy (in)equality constraints.

Yang and Cao [39], as well as Wu [36] considered the problem where the objective function in (1) becomes $Z = \vee_{i=1}^{m}(c_i \wedge x_i^{r_i})$ and the constraint parts are fuzzy relational inequalities with max–min composition. Where the objective function in (1) becomes $Z = \vee_{i=1}^{m}(c_i \cdot x_i^{r_i})$ and the constraint parts are fuzzy relational inequalities with max-product composition, the results by Zhou and Ahat [41] inspired the current paper. Yang and Cao [40] discussed the following monomial geometric programming with fuzzy relation equation constraints:

$$\min \quad z = c\prod_{j=1}^{n} x_j^{r_j}$$
$$s.t. \quad A \circ x = b.$$

Shivanian and Khorram [32] proposed monomial geometric programming subject to fuzzy relation inequalities with max-product composition.

We aim to establish an optimization management model in a BitTorrent-like Peer-to-Peer (P2P) resource sharing system. Suppose there exist n terminals in the system, denoted by A_j, $j \in J = \{1, 2, \ldots, n\}$. According to the BitTorrent-like P2P transmission protocol, every terminal shares an owner resource to the other ones and meanwhile it can download resource from any other terminals. Let the quality level when the jth terminal sends the resource data to the other terminals be x_j, $j \in J$. The bandwidth between A_i and A_j is a_{ij}. Due to the bandwidth limitation, the network traffic that A_i receives from A_j is actually $a_{ij} \wedge x_j$, where $i \in I$, $j \in J$ and $i \neq j$. When $i = j$, we always assume that $a_{ii} = 0$ since A_i does not need to download resource from itself. Furthermore,