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A distance-based framework to deal with ordinal and additive inconsistencies for fuzzy reciprocal preference relations

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ABSTRACT

Consistency of preference relations is related to rationality, the lack of consistency in decision making often leads to misleading solutions. This paper studies the ordinal and cardinal consistency problems of fuzzy reciprocal preference relations. First, the concept of ordinal consistency of fuzzy reciprocal preference relations is introduced and analyzed. A l_p distance-based method is proposed to formulate the underlying optimization problems as goal programming (GP) models for ordinal and additive consistency problems respectively. By setting different p, three GP models are obtained: (a) a linear GP model where $p = 1$; (b) a MINMAX GP model, where $p = \infty$; and (c) an extended GP model, which integrates the two previous models in particular cases. Utilizing these models, we can solve the ordinal and additive consistency problems for fuzzy reciprocal preference relations. The proposed model can preserve the initial preference information as much as possible. Finally, a numerical example and comparative analysis are provided to show effectiveness and validity of the proposed method.
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1. Introduction

Preference relations are usually used to model experts' preferences in decision making problems. The pairwise comparison method is a powerful tool and may be used to acquire the decision makers' preference values [3]. However, the experts may not have the global perception of alternatives, and thus, provide inconsistent preferences.

Consistency of preference relations is related to rationality, the lack of consistency in decision making often leads to inconsistent conclusions [16]; that is why it is important, if not crucial, to study conditions under which consistent preference relations can be obtained [12]. Shortly, we discuss the two well-known kinds of consistencies, ordinal and cardinal consistencies:

- Ordinal consistency is based on the notion of transitivity, means that if A is preferred to B and B is preferred to C, it perceives A to be preferred to C, which is normally referred as weak transitivity [4]. In crisp context, the ordinal consistency has traditionally been defined in terms of acyclicity [23], and ordinal consistency degree can be measured by the number of three-way cycles in the digraph of a preference relation [33].
- Cardinal consistency is a stronger concept than ordinal consistency. In Analytic Hierarchy Process (AHP), Saaty [22] first addressed the issue of consistency, and developed the notions of perfect consistency and acceptable consistency. The concept of

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Saaty's consistency is called multiplicative consistency, and referred as cardinal consistency. For a multiplicative preference relation $A = (a_{ij})_{n\times n}$, where $a_{ii} = 1$, $a_{ij} > 0$ and $a_{ij} = 1/a_{ji}$. If $a_{ik}a_{kj} = a_{ij}$ for all i, j and k, then A is called perfectly consistent. Generally, DM's judgments are cardinally inconsistent (i.e., $a_{ik}a_{kj} \neq a_{ij}$ for some i, j and k). Saaty [22] introduced the consistency ratio CR to measure the consistency degree of A. If $CR = CI/RI < 0.1$, then A is called acceptable consistency, where $CI = (\lambda_{max} - n)/(n - 1)$, RI is the average random index based on matrix size, λ_{max} is the maximum eigenvalue of A. For the fuzzy reciprocal preference relations, the cardinal consistency mainly includes additive consistency and multiplicative consistency.

Generally, if a preference relation is cardinally consistent, it is also ordinally consistent. However, if a preference relation is ordinally consistent, it does not lead to cardinal consistency. Sometimes, it may be cardinally inconsistent. On the other hand, ordinal inconsistency always implies cardinal inconsistency, and the converse does not hold. Furthermore, if a preference relation is ordinally inconsistent, different prioritization methods yield different ordinal rankings. Therefore, the ordinal consistency could be regarded as the most important way to increase the accuracy in the decision making process. If a preference relation is only ordinally consistent, not cardinally consistent, we can rank the alternatives directly, but the priorities could not be obtained. If a preference relation is cardinally consistent, we can not only get the priorities, but also rank the alternatives.

Many types of transitivity for fuzzy reciprocal preference relations have been widely devised [6–8]. The ordinal and cardinal consistencies of fuzzy reciprocal preference relations have been received great attention. Ma et al. [19] proposed a novel method to repair the inconsistency of fuzzy reciprocal preference relations. However, their method can only repair the ordinal inconsistency of a fuzzy reciprocal preference relation with strict comparison information, which means that there exists no indifference with strict comparison information. Xu et al. [33] first introduced the concept of the ordinal consistency for fuzzy reciprocal preference relations, then proposed a procedure to compute the ordinal consistency index (OCI), and also developed an algorithm to repair the ordinal inconsistency. Wu and Xu [25] presented an individual consistency index to measure the additive consistency degree for a fuzzy reciprocal preference relation, and a consistency control process is designed to make an inconsistent reciprocal preference relation of acceptable consistency. Xia et al. [26] developed an algorithm to improve the multiplicative consistency level of a complete/incomplete fuzzy reciprocal preference relation. However, if we use Xu et al.'s [33] method to test for Wu and Xu's [25] examples, and Xu et al.'s [28] method to test for Xia et al.'s [26] examples, we will find that, although their examples pass the test for cardinal consistency, they may still contain contradictory judgments.

However, some of the above research focuses on ordinal consistency some only focus on cardinal consistency. A few studies have paid attention to the ordinal and cardinal consistencies simultaneously. In our opinion, the ordinal and cardinal consistencies are both very important for preference relations. Ordinal consistency is a usual weak transitivity condition that should be used when avoiding the expression of inconsistent opinions, and therefore becomes the minimum requirement condition that a consistent reciprocal preference relation should verify [28].

The resolution of ordinal consistency is therefore the first task in decision making, before cardinal consistency has been sought. This is the subject of the present paper. We introduce a novel approach to deal with both kinds of inconsistencies. We propose a distance-based methodology to deal with ordinal and additive inconsistencies for fuzzy reciprocal preference relations at the same time.

In order to do this, the rest of the paper is structured as follows. Section 2 gives the basic concept of a fuzzy reciprocal preference relation, the concept of ordinal consistency and some results of ordinal consistency. In Section 3, a distance-based method is developed to deal with the ordinal consistency problem of a fuzzy reciprocal preference relation. The method is extended to cope with the additive consistency problem for a fuzzy reciprocal preference relation in Section 4. Section 5 presents a mixed analysis where firstly the ordinal consistency problem is treated, and then the additive consistency problem is solved. A numerical example is provided to show the effectiveness and validity of the proposed method, and a comparative analysis with the existing methods is provided. Finally, concluding remarks, some characteristics, and advantages of the proposed method are pointed out in Section 6.

2. Ordinal consistency of fuzzy reciprocal preference relations

For simplicity, we denote $N = \{1, 2, \ldots, n\}$. Let $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$ be a finite set of alternatives, where x_i denotes the ith alternative. In the multiple attribute decision making problems, the decision maker needs to rank the alternatives $x_1, x_2, \ldots, x_n$ from the best to the worst according to the preference information.

Definition 1 ([24]). A fuzzy reciprocal preference relation R on a set of alternatives X is represented by a fuzzy set on the product set $X \times X$, which is characterized by a membership function

$$u_R : X \times X \to [0, 1]$$

When the cardinality of X is small, the preference relation may be conveniently represented by an $n \times n$ matrix $R = (r_{ij})_{n\times n}$, where $r_{ij} = u_R(x_i, x_j)$, $\forall i, j = 1, 2, \ldots, n$. r_{ij} denotes the preference degree of the alternative x_i over x_j. Specifically, $0 \leq r_{ij} < 0.5$ denotes that x_j is preferred to x_i, and the smaller the r_{ij}, the greater the preference degree of the alternative x_j over x_i. In particular, $r_{ij} = 0$ denotes that x_j is absolutely preferred to x_i, $r_{ij} = 0.5$ denotes indifference between x_i and x_j. $0.5 < r_{ij} \leq 1$ denotes that x_i is preferred to x_j, and $r_{ij} = 1$ denotes that x_i is absolutely preferred to x_j.

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