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The optimal group continuous logarithm compatibility measure for interval multiplicative preference relations based on the COWGA operator

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ABSTRACT

The calculation of compatibility measures is an important technique employed in group decision-making with interval multiplicative preference relations. In this paper, a new compatibility measure called the continuous logarithm compatibility, which considers risk attitudes in decision-making based on the continuous ordered weighted geometric averaging (COWGA) operator, is introduced. We also develop a group continuous compatibility model (GCC Model) by minimizing the group continuous logarithm compatibility measure between the synthetic interval multiplicative preference relation and the continuous characteristic preference relation. Furthermore, theoretical foundations are established for the proposed model, such as the sufficient and necessary conditions for the existence of an optimal solution, the conditions for the existence of a superior optimal solution and the conditions for the existence of redundant preference relations. In addition, we investigate certain conditions for which the optimal objective function of the GCC Model guarantees its efficiency as the number of decision-makers increases. Finally, practical illustrative examples are examined to demonstrate the model and compare it with previous methods.

1. Introduction

Group decision-making (GDM) is a common task in human activities that consists of determining the most preferred alternative(s) from a given set of possibilities, as performed by a group of decision-makers (DMs). In recent decades, GDM with preference relations has received considerable attention [3,5,7,9,28]. However, because of time pressure, lack of knowledge and limited expertise, the preference relations provided by DMs are typically given in the form of numerical intervals rather than exact values. Saaty and Vargas [17] formulated the concept of interval multiplicative preference relations, which are a very important type of preference relation in the GDM process. In the literature, we find a wide range of techniques developed to address interval multiplicative preference relations [6,8,24].

The key to GDM with interval multiplicative preference relations is to know how to effectively aggregate all of the individual preference relations, which involves two procedures. The first is to determine whether all of the individual preference relations

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can be aggregated, which is achieved through consistency and compatibility. Consistency plays a role in ensuring that the interval multiplicative preference relations are neither random nor illogical in their pairwise comparisons [4,19,25,33]. A lack of consistency in decision-making with interval multiplicative preference relations leads to inconsistent conclusions. Many consistency techniques have been suggested for interval multiplicative preference relations [6,8,24,33]. Compatibility ensures that all of the individual preference relations can be effectively aggregated. In GDM, the ordering of alternatives by a group will always be inconsistent with the ordering preferred by each of the participating individuals because of the transitivity of preference [15]. To analyze how individuals develop expectations about the consensus of their rankings with that of a group to which they provide input, experts always assume that each individual performs a complete ranking of the alternatives and compares it with the group ranking. Compatibility is then defined as a measure of the consensus of rankings between the group and each individual. When such a compatibility measure is used, inconsistency is then allowed for an individual or a group, and they can adjust their judgments and incompatibility up to a certain tolerance level. This results in a lack of acceptable compatibility and leads to unsatisfactory decision-making with preference relations.

Saaty [15] was the first to discuss the compatibility of preference relations. Xu [29] evaluated the compatibility of interval fuzzy preference relations based on a distance measure. Chen, Zhou and Han [2] developed a measure of the compatibility of interval fuzzy linguistic preference relations. Wu, Cao and Zhang [26] proposed a means of assessing the compatibility of interval multiplicative preference relations.

However, the compatibility measures mentioned above are all defined based on the endpoints of numerical intervals, which means that these compatibility measures have uniform distributions on the corresponding interval preference relations. Obviously, this scenario is different from that of GDM problems in uncertain environments. It is necessary to accurately consider the risk attitudes of DMs. To address this issue, continuous interval information aggregation operators are employed to cope with interval values. One well-known continuous interval information aggregation operator is the continuous ordered weighted averaging (COWA) operator introduced by Yager [31], which is based on the ordered weighted averaging (OWA) operator [30]. It provides a parameterized family of aggregation operators that includes both the maximum and minimum. Since its emergence, the COWA operator has been attracting increasing attention from both decision-makers and researchers [27,32,34,35]. Moreover, inspired by the COWA operator, Yager and Xu [32] introduced the continuous ordered weighted geometric averaging (COWGA) operator, which is used to cope with interval multiplicative preference relations.

The second necessary procedure in GDM with interval multiplicative preference relations is to quantify the DMs' weights, which is an interesting and important research topic in GDM. Various models have been developed to determine the DMs' weights based on interval multiplicative preference relations [1,8,24,26,27]. For example, Wu et al. [27] proposed two information aggregation operators to determine the weights of DMs, including the reliability induced continuous ordered weighted geometric averaging operator, which orders the argument values based upon the reliability of the information sources, and the relative consensus degree induced continuous ordered weighted geometric averaging operator, which orders the argument values based on the relative consensus degree of the information sources. Wang, Yang and Xu [24] introduced a nonlinear programming approach to generating interval weights based on the consistency of interval multiplicative preference relations. Liu, Zhang and Wang [8] developed a goal programming model to determine the weighting vector of incomplete interval multiplicative preference relations based on an interval weighted geometric averaging operator. These measures are calculated from the relative projections of the individual preference relations onto the collective preference relation by extending the relative projections of the vectors. Chen and Zhou [1] defined a consensus indicator for interval multiplicative preference relations to determine the weights of the DMs in GDM based on relative entropy. In the studies listed above, the consistency property is always used to obtain the experts' weighting vector; in other words, the compatibility measure is neglected. Additionally, Wu, Cao and Zhang [26] developed the induced continuous ordered weighted geometric averaging operator to determine expert weights based on the compatibility indices of DMs with interval multiplicative preference relations. However, this method is associated with the basic unit monotonic function [30], which means that compatibility is not truly used to determine the experts' weighting vector.

The objective of this paper is to develop a new compatibility measure for interval multiplicative preference relations based on the COWGA operator and to determine the optimal weights of DMs by constructing a group continuous compatibility model (GCC Model). We present a new compatibility measure called the continuous logarithm compatibility measure for interval multiplicative preference relations, in which the risk attitudes of the DMs are considered using the COWGA operator. We study several desirable properties of the compatibility measure and investigate the relationship between each DM's synthetic interval multiplicative preference relation and the continuous characteristic preference relation.

To determine the weights of the DMs in group decision-making with interval multiplicative preference relations, we construct our GCC Model by minimizing the group continuous logarithm compatibility measure between the synthetic interval multiplicative preference relation and the continuous characteristic preference relation. Next, the sufficient and necessary for the existence of an optimal solution are studied. Further attention is also given to the conditions for the existence of a superior optimal solution and of redundant preference relations in the GCC Model. Moreover, we investigate several properties of the GCC Model by adding additional DMs into the GDM process.

The remainder of the paper is organized as follows. In Section 2, we briefly describe several preliminaries. Section 3 presents the new compatibility measure for interval multiplicative preference relations, and we develop the GCC Model to determine the DMs' weights and investigate certain properties of the GCC Model. Section 4 provides an illustrative example, and in Section 5, we conclude the paper by summarizing the main conclusions.