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Robustness analysis for decision under uncertainty with rule-based preference model[☆]

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ABSTRACT

We consider decision under uncertainty as a multi-attribute classification problem where a set of acts is described by outcomes gained with given probabilities. The Decision Maker (DM) provides desired classification for a small subset of reference acts. Such preference information is structured using Dominance-based Rough Set Approach (DRSA), and the resulting lower approximations of the quality class unions are used as an input for construction of an aggregate preference model. We induce all minimal-cover sets of rules being compatible with the non-ambiguous assignment examples, and satisfying some additional requirements that may be imposed by the DM. Applying such compatible instances of the preference model on a set of all acts, we draw conclusions about the certainty of recommendation assured by different minimal-cover sets of rules. In particular, we analyze the diversity of class assignments, assignment-based preference relations, and class cardinalities. Then, we solve an optimization problem to get a univocal (precise) classification for all acts, taking into account the robustness concern. This optimization problem admits incorporation of additional indirect and imprecise preferences in form of desired class cardinalities and assignment-based pairwise comparisons. Finally, we extend the proposed approach to group decision under uncertainty. We present a set of indicators and outcomes giving an insight into the spaces of consensus and disagreement between the DMs.

1. Introduction

Decision under uncertainty is a classical topic of decision theory (see [11] for a review). In this case, the Decision Maker (DM) considers a set of acts whose consequences are uncertain. There are many possible states of the world with given probabilities. Depending on the actual state, an act can yield a corresponding outcome with a given probability. It is assumed, moreover, that the DM is able to express preferences with respect to the outcomes predicted for the considered acts with given probabilities. The preference information provided by the DM takes part in the construction of her/his preference model. This model induces

☆ This paper significantly extends [37] which won the Best Paper Award at the 2014 Joint Rough Set Symposium (JRS 2014).

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a preference relation in the set of acts, richer than the stochastic dominance relation [44]. Its proper exploitation leads to a recommendation in terms of choice, ranking, or classification.

The main approaches to modeling decision under uncertainty are based on the expected utility theory, which was axiomatized by von Neumann and Morgenstern [43] for objective probability, and by Savage [33] for subjective probability. Many experiments showed, however, systematic violation of the expected utility hypotheses (see, e.g., [1,10,26]). In consequence, many alternative models weakening some original axioms have been proposed (see [38] for a review). In this context, it is worth mentioning the work on ambiguity as a source of uncertainty [29]. Furthermore, Greco, Matarazzo, and Słowiński [16] proposed an approach to decision under uncertainty based on stochastic dominance, which is the weakest assumption possible. They adapted for this their Dominance-based Rough Set Approach (DRSA) [14], which extends the rough set concept introduced by Pawlak [30] by handling ambiguity caused by violation of dominance (for some recent advances in DRSA, see, e.g., [22,28,36,39,40,45]).

In the approach presented in [16], the decision under uncertainty is formulated in terms of a multi-attribute classification problem. The set of classified objects is the set of acts and the set of condition attributes describing the acts is the set of probabilities derived from an additive probability distribution defined over disjoint and exhaustive states of the world. The method expects the DM to assign a small subset of acts, called *reference acts*, to some pre-defined classes of overall quality. Precisely, each assignment of a reference act to a quality class is a classification example characterized by the outcomes for a finite set of given probabilities (condition attributes), and by the assignment to one of several classes of overall quality (decision attribute). The set of classification examples constitutes preference information conditioned by the value system of the DM. Such classification data is structured using DRSA, and then a set of decision rules is induced. These rules explain the preferences of the DM in terms of conditions on values of outcomes for particular probabilities, that ensure assignment of reference acts to the so-called upward union or downward union of classes. A set of rules covering the classification examples constitutes a preference model of the DM. It is subsequently used to classify the non-reference acts. An intuitive principle which guides this approach can be formulated as follows: “the more and the more probable, the better”.

When applying DRSA to decision under uncertainty in the way suggested in [16], one needs to be aware of an important limitation of this approach. When classifying the non-reference acts, the authors of [16] use just a single set of rules. However, such representation of the DM preferences is not unique, because, in general, there may exist many sets of rules that are compatible with the provided preference information. Thus, choosing among them is to a large extent arbitrary. Moreover, when applied on the non-reference acts, different sets of rules may suggest different class assignments for the same acts. As explained in [6], existence of alternative instances of a preference model - all of them being compatible with the input preference information - requires robustness analysis of the recommendations delivered by all these instances.

Addressing the above mentioned drawback, we proposed a two-fold revision of DRSA to decision under uncertainty [37]. On the one hand, we considered all minimal-cover sets of rules as compatible instances of the preference model. Adopting this strategy, we avoided arbitrary selection of one among many sets of rules which reproduce equally well the provided preference information (classification examples). On the other hand, we investigated the diversity of the recommendations suggested by these sets by producing two types of assignment for each act. The *possible assignment* holds if and only if it is confirmed by at least one compatible minimal-cover set of rules, whereas the *necessary assignment* needs to be supported by all minimal-cover sets of rules. In this way, we adapted a more general principle of Robust Ordinal Regression (ROR) [6] to decision under uncertainty formulated as a multi-attribute classification problem. The aim of this paper is to extend this approach by proposing some advanced methods for robustness analysis in the context of decision under uncertainty (for other applications of ROR to ordinal classification (sorting) problems, see [18,24,25]).

First, we adapt an integrated framework for robustness analysis in multiple criteria sorting problem [20] to DRSA. Thus, when investigating the stability of the delivered recommendation, apart from considering class assignments for each act individually, we take into account two other perspectives. On the one hand, we refer to the assignment-based preference relation which holds for an ordered pair of acts if one of them is assigned to a class at least as good as the other [13,24]. Such a relation is called an assignment-based outranking relation. On the other hand, we compute cardinalities of the quality classes. The basic analysis consists in considering the necessary, possible, or extreme outcomes. However, since all compatible minimal-cover sets of rules are known, we are able to compute the assignment-based outranking indices or class cardinality indices defined as the shares of compatible minimal-cover sets of rules which confirm some classification result. Overall, the DM may observe the impact of her/his preference information on the recommendation concerning not only the whole set of acts, but also all pairs of acts and all quality classes.

Second, we exploit the results of robustness analysis to construct a univocal recommendation that would suggest assignment of each act to a single class. For this purpose, we analyze the cumulative class acceptability indices reflecting the shares of minimal-cover sets of rules that assign an act to each class. A natural proposal consists in selecting for each act a class with the maximal acceptability. However, we extend this basic proposal by accounting for additional types of indirect and imprecise preference information to be taken into account when constructing a univocal recommendation. These are desired class cardinalities and assignment-based pairwise comparisons. The former specify the minimal and/or maximal number of acts that can be assigned to each class [23], while the latter indicate imprecise comparison between desired classes for pairs of acts, but without specifying any concrete classes [20]. Note that the proposed procedure is more general, being independent of the method used for robustness analysis. Thus, it is equally desirable for use with some value- [24] or outranking-based [41] stochastic approaches to Multiple Criteria Decision Aiding (MCDA).

Third, we extend DRSA for decision under uncertainty to group decision [4,15]. In this approach, each DM provides her/his individual preference information. Then, the collective results account for the robustness analysis conducted for each DM