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A model for type-2 fuzzy rough sets

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A R T I C L E I N F O

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A B S T R A C T

Rough set theory is an important approach to granular computing. Type-1 fuzzy set theory permits the gradual assessment of the memberships of elements in a set. Hybridization of these assessments results in a fuzzy rough set theory. Type-2 fuzzy sets possess many advantages over type-1 fuzzy sets because their membership functions are themselves fuzzy, which makes it possible to model and minimize the effects of uncertainty in type-1 fuzzy logic systems. Existing definitions of type-2 fuzzy rough sets are based on vertical-slice or α-plane representations of type-2 fuzzy sets, and the granular structure of type-2 fuzzy rough sets has not been discussed. In this paper, a definition of type-2 fuzzy rough sets based on a wavy-slice representation of type-2 fuzzy sets is given. Then the concepts of granular type-2 fuzzy sets are proposed, and their properties are investigated. Finally, granular type-2 fuzzy sets are used to describe the granular structures of the lower and upper approximations of a type-2 fuzzy set, and an example of attribute reduction is given.

1. Introduction

Since the theory of type-1 fuzzy sets was proposed by Zadeh in 1965 [21], it has been widely used in many areas of artificial intelligence. However, according to Mendel [10], there exist at least four sources of uncertainty in type-1 fuzzy logic systems: (1) the meanings of the words used in rule antecedents and consequents may be uncertain (the same word may mean different things to different people); (2) the consequents may have a histogram of values associated with them, especially when knowledge is extracted from a group of experts, all of whom do not agree with each other; (3) the measurements that activate a type-1 fuzzy logic system may be noisy and therefore uncertain; and (4) the data used to tune the type-1 fuzzy logic system parameters may also be noisy. All these uncertainties lead to uncertain fuzzy set membership functions.

Type-2 fuzzy sets were introduced by Zadeh [22] as an extension of the concept of type-1 fuzzy sets (ordinary fuzzy sets). Type-2 fuzzy sets can be used to describe the four kinds of uncertainty listed above because the membership functions of type-2 fuzzy sets are themselves fuzzy. However, type-2 fuzzy sets are nowhere near as widely used as type-1 fuzzy sets.

The secondary membership functions for general type-2 fuzzy sets are too difficult to construct and too complex to compute, and only a special kind of type-2 fuzzy set, the interval type-2 fuzzy set, is widely used because its secondary membership grades are equal to one. In other words, the membership degree of every element in the universe is characterized by a sub-interval of [0, 1], and any of the values in the sub-interval can be assigned as the membership degree, with each value having the same

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probability. However, in practice, the probabilities of the values in the sub-interval may follow another distribution, such as the normal or triangular distribution [9].

In 1982, Pawlak proposed the theory of rough sets as a new mathematical tool for reasoning about data. Rough set theory has been under development for thirty years and has been successfully used in various fields of artificial intelligence such as expert systems, machine learning, pattern recognition, decision analysis, process control and knowledge discovery in databases [15]. Traditional rough set theory is based on an equivalence relation, which seems to be a very restrictive condition that may limit the applications of the rough set model. For example, the values of attributes may be both symbolic and real-valued, in which case they cannot be manipulated by traditional rough set theory. Two close values may differ only as a result of noise, but in traditional rough set theory, they may be considered to be of different orders of magnitude. To overcome these shortcomings, Dubois and Prade [4] combined fuzzy sets and rough sets by proposing definitions of rough fuzzy sets and fuzzy rough sets in 1990, after which many studies were carried out in the field of fuzzy rough sets. Shen and Jensen [18] proposed an approach that integrates a fuzzy rule induction algorithm with a fuzzy rough method for feature selection. Jensen and Shen [5] provided an interval-valued approach for fuzzy rough feature selection, which could handle missing values and uncertainties that could not be modeled by a type-1 approach. Wu et al. [20] proposed an attribute reduction method within the interval type-2 fuzzy rough set framework and presented the properties of interval type-2 fuzzy rough sets. To date, most research in fuzzy rough sets has been restricted to ordinary (type-1) fuzzy environments and interval type-2 fuzzy environments [1,3,6,7,17,23–25,27,28].

The point-valued representation is usually the starting point for understanding or describing a general type-2 fuzzy set, but it does not seem to be useful for much of anything else [13]. In addition, there are three other popular representations for a type-2 fuzzy set: the vertical-slice representation, the wavy-slice representation (which is also called the Mendel-John representation or an embedded type-2 fuzzy set representation), and the α-plane representation. Based on vertical-slice representations of type-2 fuzzy sets, Wang [19] investigated type-2 fuzzy rough sets on two finite universes of discourse using both constructive and axiomatic approaches and discussed the topological properties of type-2 fuzzy rough sets. Using α-plane representation theory, Zhao and Xiao [26] presented definitions of general type-2 fuzzy rough sets and studied some basic properties of upper and lower approximation operators. In addition, they examined the connections between special general type-2 fuzzy relations and general type-2 fuzzy rough upper and lower approximation operators and characterized various classes of general type-2 fuzzy rough approximation operators using an axiomatic approach. Many properties were proposed in [19] and [26], but no discussion of the granular structure of type-2 fuzzy rough sets was included.

According to Pedrycz, “Information granules are intuitively appealing constructs, which play a pivotal role in human cognitive and decision-making activities. We perceive complex phenomena by organizing existing knowledge along with available experimental evidence and structuring them in a form of some meaningful, semantically sound entities, which are central to all ensuing processes of describing the world, reasoning about the environment, and supporting decision-making activities” [16]. In classical rough set theory, lower and upper approximations are defined as unions of certain sets, exhibiting a clear granular structure over sets. Chen et al. [2] proposed the concept of granular fuzzy sets based on fuzzy similarity relations and described the granular structures of the lower and upper approximations of a fuzzy set within the framework of granular computing. The wavy-slice representation of a type-2 fuzzy set in terms of embedded type-2 fuzzy sets is most valuable in theoretical studies because it quickly leads to the structure of the solution to a new problem. To discuss the granular structure of type-2 fuzzy rough sets, a model for type-2 fuzzy rough sets is proposed here using the wavy-slice representation of a type-2 fuzzy set presented by Mendel and John [12]. Here the conclusions of Chen et al. are extended by proposing the concept of granular type-2 fuzzy sets and investigating their properties. Then these granular type-2 fuzzy sets are used to describe the granular structures of the lower and upper approximations of a type-2 fuzzy set. Finally, an example of attribute reduction within a type-2 fuzzy rough framework is presented.

The rest of this paper is organized as follows. Fundamental concepts and properties that will be used in this paper are reviewed in Section 2. Section 3 introduces the definition of a type-2 fuzzy rough set based on the wavy-slice representation. In Section 4, the granular structure of type-2 fuzzy rough sets is discussed using granular type-2 fuzzy sets. Conclusions are presented in Section 5.

2. Preliminaries

This section will review some basic notions and properties related to type-2 fuzzy sets, rough sets, and fuzzy rough sets.

2.1. Type-2 fuzzy sets

Definition 1 [12]**.** Let X be a nonempty universe of discourse. A type-2 fuzzy set, $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$, i.e.,

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | x \in X, u \in J_x \subseteq [0, 1]\},$$

in which $0 \le \mu_{\tilde{A}}(x, u) \le 1$. $\tilde{A}$ can also be expressed as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u)/(x, u), J_x \subseteq [0, 1], \tag{1}$$

where $\int\int$ denotes union over all admissible x and u. The class of all type-2 fuzzy sets of the universe X is denoted by $\tilde{F}(X)$.