Information Sciences 328 (2016) 485–509

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Obstacles and difficulties for robust benchmark problems: A novel penalty-based robust optimisation method

Seyedali Mirjalili [a,b,*], Andrew Lewis [a]

[a] School of Information and Communication Technology, Griffith University, Nathan, Brisbane, QLD 4111, Australia
[b] Griffith College, Mt Gravatt, Brisbane, QLD 4122, Australia

ARTICLE INFO

Article history:
Received 6 October 2014
Revised 16 May 2015
Accepted 3 August 2015
Available online 28 August 2015

Keywords:
Optimization
Test problem
Robustness
Particle swarm optimization
Benchmark
Algorithm

ABSTRACT

This paper first identifies a substantial gap in the literature of robust optimisation relating to the simplicity, low-dimensionality, lack of bias, lack of deceptiveness, and lack of multi-modality of test problems. Five obstacles and difficulties such as desired number of variables, bias, deceptiveness, multi-modality, and flatness are then proposed to design challenging robust test problems and resolve the deficiency. A standard test suit of eight robust benchmark problems is proposed along with controlling parameters that allow researchers to adjust and achieve the desired level of difficulty. After the theoretical analysis of each proposed test function, a robust particle swarm optimisation (RPSO) algorithm and a robust genetic algorithm (RGA) are employed to investigate their effectiveness experimentally. The paper also inspects the effects of the proposed controlling parameters on the difficulty of the test problems and the proposal of a novel penalty function to penalize the solutions proportional to their sensitivity to perturbations in parameters. The results demonstrate that the proposed test problems are able to benchmark the performance of robust algorithm effectively and provide different, controllable levels of difficulty. In addition, the comparative results reveal the superior performance and merits of the proposed penalty-based method in finding robust solutions. *Note*: The source codes of the proposed robust test functions are publicity available at www.alimirjalili.com/RO.html.

1. Introduction

Optimisation using meta-heuristics has become very popular over the last two decades. Such optimisation techniques are considered as stochastic methods, which approximate the global optima for optimisation problems. They mostly make decisions and approximate the global optimum by providing inputs for optimisation problems and observing their outputs. For instance, the particle swarm optimisation (PSO) [10,21] finds the best inputs with the best fitness value over the course of iterations and uses them for improving other solutions. Due to gradient-free mechanism, low probability of local optima stagnation, and high flexibility, the applications of meta-heuristics can be found in different fields of science and industry [6,41].

A meta-heuristic may face several difficulties when approximating the global optimum for a given real problem. Due to the unknown shape of the search space of real problems, however, it is impossible to observe the characteristics of search spaces and consequently the behaviour of meta-heuristics directly. Therefore, researchers typically employ test functions when developing

* Corresponding author at: School of Information and Communication Technology, Griffith University, Nathan, Brisbane, QLD 4111, Australia. Tel.: +61434555738.
E-mail address: ali.mirjalili@gmail.com, seyedali.mirjalili@griffithuni.edu.au (S. Mirjalili).

http://dx.doi.org/10.1016/j.ins.2015.08.041

[2,20], improving [37,38,67,73,75,83–92], or analysing [25,65] an algorithm in this field. The so-called benchmark or test problems mostly refer to the mathematical functions that have known search spaces and are intended to simulate the characteristics of real problems.

Some of the most important difficulties involved in solving real problems are: slow convergence, large number of local optima, large number of variables, dependency of variables, constraints, deceptive search spaces, flat search spaces, and uncertainties. The literature shows that researchers developed different benchmark problems for mimicking each of these characteristics of real search spaces.

For benchmarking the convergence rate of an algorithm, unimodal test functions have been designed and employed. A unimodal test function has a single global optimum and there are no other local optima in the search space. In addition, the entire search space favours this single optimum, which makes it suitable for benchmarking the convergence speed of meta-heuristics. In the literature, there are many unimodal test functions of which the most popular ones are: sphere, Easom, Beale's function, Matyas, and McCormick. Unimodal test functions benchmark not only convergence speed but also exploitation of a meta-heuristic.

The second difficulty, large number of local optima, are implemented by multi-modal [79] and composite test functions [36,76]. As its names implies, a multi-modal test function has multiple optima of which one of them is the global and the rest are local. Such test functions are designed mostly with an exponentially growing number of local optima proportional to the number of variables. Some of the most popular multi-modal test functions in the literature are: Ackley, Schaffer, Griewank, Rastrigin, Michalewicz, and Schwefel. Composite test functions also belong to the family of multi-modal test functions, which are constructed by combining, shifting, rotating, and biasing of other unimodal and multi-modal test functions. Multi-modal and composite test functions provide a large number of local optima so that the performance of a meta-heuristic can be benchmarked in terms of local optima avoidance and exploration.

The difficulty of a problem is increased proportional to the number of variables as well. A large number of variables is usually an intrinsic characteristic of the test functions in the literature. Therefore, a desired number of variables can easily be obtained by the summation or multiplication of the parameters. Another difficulty here is that the variables may have dependencies (linkage) in real problems. In the literature there are some works to simulate such difficulties as well [17,80].

Deceptive and flat search spaces are two special characteristics that may occur when solving a real problem. In the former case, the entire search space provides deceptive and misleading information about the location of the global optimum. In the latter case, the search space provides very little and sometimes no information about the possible location of the global optimum. Both of these cases provide very challenging test beds for meta-heuristics and may be found in [15,94].

Last but not least, uncertainties are very critical undesirable inputs for optimisation problems. Uncertainties can occur in inputs, outputs, constraints, and operating conditions of optimisation problems [5,30,68]. The process of considering any of these uncertainties during optimisation is called robust optimisation. Robust optimisation allows us to make sure that the objective value of the final optimal solution will stay stable in case of uncertainties at maximum level. For benchmarking the performance of robust algorithms, there are different test functions in the literature as well [7,19,32,34]. However, there are several drawbacks that should be resolved.

For mimicking uncertainties in parameters, which is the focus of this paper, there is very little in the literature. In addition, the majority of the current robust test problems suffer from simplicity, low-dimensionality, lack of bias, lack of deceptiveness, and lack of a large number of local/non-robust optima. This motivates our attempts to provide an empirical study of the suitability of the current robust benchmark problems, improve their difficulties by five different types of obstacles, and propose a set of challenging robust test problems for benchmarking the performance of robust algorithms from different perspectives. The rest of the paper is organised as follows.

Section 2 provides the concepts of robust optimisation and different techniques to handle uncertainties in parameters. The characteristics of the current robust test problems and their drawbacks also are investigated in this section. The obstacles are presented and integrated into some of the current test functions in Section 3. This section also considers the proposal of other, new robust test problems as well as a novel penalty-based robust optimisation approach. Section 4 employs a particle swarm optimisation (PSO) and a genetic algorithm (GA) for first empirically proving the simplicity and ineffectiveness of the current robust test problems. These two algorithms are then compared on the proposed set of test functions to prove their effectiveness in practice. The comparative results of the proposed penalty-based algorithms are presented in Section 4 as well. Finally, the conclusions and future works are discussed in Section 5.

2. Robust optimisation

There are different types of uncertainties, but the most common types are manufacturing errors and production perturbations. This work concentrates on this type of uncertainty. In this type, the variables of a particular problem may vary after finding the optimum. This undesired fluctuation might degrade the output of the cost function significantly. A real example of such perturbations is the resolution of manufacturing devises such as CNC machinery when building different parts of a system. In this case, design variables may vary by the maximum resolution of the production machinery and negatively impact the desired inputs of the system. Without loss of generality, a robust optimisation problem with respect to the perturbation in the variables (parameters) is formulated as follows [5,30]:

$$Minimise: \quad F(\vec{x}+\vec{\delta}) \tag{2.1}$$