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Relationships between generalized rough sets based on covering and reflexive neighborhood system

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ABSTRACT

Rough set theory is an important tool to deal with uncertainty, granularity and incompleteness of knowledge. The covering-based rough sets is one of the most important extensions of the classical Pawlak rough sets. In covering-based rough set theory, many types of rough sets were already defined in the literature. To find out the relationships among different types of covering-based rough sets and give an axiomatic definition is a very important issue. The generalized rough sets in neighborhood system is another important extension of the classical Pawlak rough sets. In this paper, we investigate relationships between covering-based rough sets and generalized rough sets in neighborhood system. Then, twenty-three types of the element based, the granule based and the subsystem based definitions of covering approximation operators are unified under the framework of generalized approximation operators in neighborhood systems. Moreover, properties of the generalized approximation operators in neighborhood systems are presented.

1. Introduction

Rough set theory, proposed by Pawlak [22], is a useful mathematical tool to deal with uncertainty, granularity and incompleteness of knowledge. Rough set theory has been widely used in feature selection, rule extraction, uncertainty reasoning, decision evaluation, granular computing and so on [5,10,23,24,26,38,51]. In the Pawlak rough set model, the lower and upper approximation operators based on equivalence (indiscernibility) relations or partitions are two important basic concepts. However, the requirement of equivalence relation may limit the application domain of the rough set model. Therefore, many extensions of the classical Pawlak rough set have been proposed, which are based on more general binary relations [4,33,34,45,46], coverings [1–3,7,18,43,50,54], fuzzy sets [8,9,19,20,28,29,35,36,40–42], neighborhood systems [12–14,21,49] and general approximation framework [6].

The covering-based rough sets is one of the most important extensions of the classical Pawlak rough sets. In the covering-based rough set theory, various kinds of rough sets were already defined and studied in the literature. For instance, Zakowski first extended the Pawlak rough set model from partition to covering [50]. Pomykala investigated some properties of two pairs of dual covering approximation operators within topological spaces [25]. Xu and Zhang proposed a pair of covering approximation operators and defined a measure of roughness with this pair of lower and upper approximation operators [44]. Zhu introduced a type of covering-based rough sets from the topological view and explored topological

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properties of the type of rough sets [54]. Tsang et al. defined a new covering upper approximation operator in [37]. There have also been much more contributions in the study of relevance among different types of covering approximation operators. Zhu investigated relationships among six types of covering-based rough sets [53]. Liu and Sai gave a comparison of two types of rough sets induced by coverings [17]. Qin et al. discussed some properties of five pairs of dual covering approximation operators, and presented conditions with which these covering approximation operators are identical [27]. Yao and Yao proposed a framework for the study of covering approximation operators by the element based, the granule based and the subsystem based definitions [48]. Zhang and Luo changed five pairs of covering approximation operators into the same pair of relation approximation operators [52]. Liu discussed relationships among four types of covering-based rough sets [16]. Restrepo et al. investigated partial order relation for approximation operators in covering based rough sets [30]. Restrepo et al. used the concepts of duality, conjugacy and adjointness to establish relationships between the most commonly used covering approximation operators [31].

The generalized rough sets in neighborhood system is another important extension of the classical Pawlak rough sets. It attracts the attention of researchers much more. Lin and Michael explored neighborhood systems and approximations in neighborhood systems [12,21]. Approximation retrieval and information retrieval were summarized in [13,14]. Yao also did some researches about neighborhood systems and approximate retrieval models based on neighborhood systems [49]. Wang et al. investigated relationships between generalized rough sets in six coverings and pure reflexive neighborhood system [39].

From the introduction above, we can see that the generalized rough sets based on covering and neighborhood system attract much interests of researchers. Then it is natural to explore connections and differences between the two generalized rough sets to develop the theories and applications of these two generalized rough sets. In this paper, we point out that the element based, the granule based and the subsystem based definitions of covering-based rough sets in [48] can be changed into the generalized rough sets in neighborhood systems. In the next section, we present the definition of a pair of generalized approximation operators in neighborhood system, and explore some properties of the approximation operators. We also establish different models of neighborhood systems and discuss their properties. In Section 3, we change twenty-three types of covering approximation operators into the generalized approximation operators in different neighborhood systems, and explore properties of the approximation operators in neighborhood systems.

2. Generalized rough sets in neighborhood system

In this section, we introduce some basic concepts about neighborhood system and generalized rough sets in neighborhood system.

Definition 1 [48]. Let U be the universe of discourse, and $\mathcal{P}(U)$ be the class of all subsets of U. A mapping $n : U \rightarrow \mathcal{P}(U)$ is called a neighborhood operator. For any $x \in U, n(x) \in \mathcal{P}(U)$ is called a neighborhood of x. If $x \in n(x)$ for all $x \in U$, then n is called a reflexive neighborhood operator.

Sierpiński introduced the notion of a Fréchet (V) Space [32], which is a set U that each element $x \in U$ is associated with a non-empty family of neighborhoods of U called neighborhood system. Lin also presented much discussion about the neighborhood system [12].

Definition 2 ([12,32]). A neighborhood system of an object $x \in U$, denoted by $NS(x)$, is a non-empty family of neighborhoods of x. The set $\{NS(x)|x \in U\}$ is called as a neighborhood system of U, and it is denoted by $NS(U)$. Let $NS(U)$ be a neighborhood system of U.

$NS(U)$ is said to be serial, if for any $x \in U$ and $n(x) \in NS(x), n(x)$ is non-empty (called Fréchet (V) Space in [32]).
$NS(U)$ is said to be reflexive, if for any $x \in U$ and $n(x) \in NS(x), x \in n(x)$.
$NS(U)$ is said to be symmetric, if for any $x, y \in U, n(x) \in NS(x)$ and $n(y) \in NS(y), x \in n(y) \Rightarrow y \in n(x)$.
$NS(U)$ is said to be transitive, if for any $x, y, z \in U, n(y) \in NS(y)$ and $n(z) \in NS(z), x \in n(y)$ and $y \in n(z) \Rightarrow x \in n(z)$.

Lin proposed the concept of neighborhood system by the relational granular model [11], which can be seen as a special case of the neighborhood system in Definition 2. Wang et al. presented the definition of lower and upper approximations in a reflexive neighborhood system defined by relational granular model [39]. Lin and Yao defined the approximation operators in a general neighborhood system [15]. In the following, we present the definition of approximation operators in a general neighborhood system.

Definition 3 [15]. Let $NS(U)$ be a neighborhood system of U. The lower and upper approximations of X are defined as follows:

$$\underline{apr}_{NS}(X) = \{x \in U | \exists n(x) \in NS(x), n(x) \subseteq X\},$$
$$\overline{apr}_{NS}(X) = \{x \in U | \forall n(x) \in NS(x), n(x) \cap X \neq \emptyset\}.$$

$\underline{apr}_{NS}(X), -\overline{apr}_{NS}(X)$ and $\overline{apr}_{NS}(X) - \underline{apr}_{NS}(X)$ are, respectively, called positive, negative and boundary regions of X, and denoted by $POS_{NS}(X), NEG_{NS}(X)$ and $BN_{NS}(X)$. X is called a definable set if $\underline{apr}_{NS}(X) = \overline{apr}_{NS}(X)$, and it is rough otherwise. Furthermore,