Information Sciences 305 (2015) 56–76

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Extending multi-objective differential evolution for optimization in presence of noise

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ARTICLE INFO

Article history:
Received 8 March 2014
Received in revised form 1 December 2014
Accepted 1 February 2015
Available online 7 February 2015

Keywords:
Multi-objective optimization
Differential evolution
Noise-handling
Type-2 fuzzy set
Stochastic selection

ABSTRACT

The paper aims at designing new strategies to extend the selection step of traditional Differential Evolution for Multi-objective Optimization algorithm to proficiently obtain Pareto-optimal solutions in presence of noise. The first strategy, referred to as adaptive selection of sample size, is employed to balance the trade-off between accurate fitness estimate and computational complexity. The second strategy is concerned with determining defuzzified centroid value of the noisy fitness samples, instead of their conventional averaging, as the fitness measure of the trial solutions. The third extension is concerned with the introduction of a probabilistic Pareto ranking strategy to tarnish the detrimental effect of noise incurred in deterministic selection of traditional algorithms. The fourth strategy attempts to extend Goldberg's approach to examine possible placement of a slightly inferior solution in the optimal Pareto front using a more statistically viable comparator. Finally, to ensure the diversity in distribution of quality solutions in the noisy fitness landscapes, a new selection criterion induced by the crowding distance measure and the probability of dominance is formulated. Experiments undertaken to study the performance of the extended algorithm reveal that the extended algorithm outperforms its competitors with respect to four performance metrics, when examined on a test-suite of 23 standard benchmarks with additive noise of three statistical distributions.

1. Introduction

Multi-Objective Optimization (MOO) refers to jointly optimizing two or more objective functions, describing functional relationships between measurement and estimator variables, of a real-world system. Because of noisy measurements, the objective estimates are also influenced by noise. Determining solutions of system variables that jointly satisfy multiple objectives in presence of noise is an important issue in the realm of optimization algorithms. These optimization problems are usually referred to as noisy multi-objective optimization [25,45].

Traditional derivative-free single objective optimization algorithms usually have three main steps: initialization, adaptation of trial solutions and selection of quality solutions to evolve through the adaptation phase until convergence is achieved. MOO algorithms differ from their single objective counterpart in the selection step. In a single objective optimization, selection is performed based on the objective estimate (also called fitness measure) of the trial solutions. Trial solutions having better fitness measure are selected to pass through the adaptation phase, and the rest of the population is dropped out to maintain a uniform population size over the generations of the algorithms. In a MOO [7], we, however, require to optimize

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http://dx.doi.org/10.1016/j.ins.2015.02.008
0020-0255/

two or more conflicting objective functions jointly. These algorithms primarily differ from their single objective counterpart by the selection strategy. Selection of trial solutions in multi-objective settings is performed in two steps. In the first step, the principle of Pareto ranking is employed to select a list of *non-dominated* (equally good) trial solutions for placement in the *Pareto fronts*. The second step includes a concept of *crowding distance* to drop a few trial solutions from the selected lowest rank Pareto front to maintain uniform population size over generations. In noisy MOO, the objective estimates being noisy, a quality trial solution may be deceived because of its poor (noisy) fitness estimates and may be discarded from the optimal Pareto front, while a deceptive solution with illusive good fitness may be promoted to the next generation [1,5].

There exist quite a few papers to extend traditional MOO algorithms to function well in presence of noise in fitness estimates [2,3,6,21–23,34,38–40]. The common strategy used to handle the problem is to utilize the concept of sampling (fitness re-evaluation of the same trial solution) [40] to improve fitness estimates in presence of noise. In [2], a statistical comparator is designed to allow the ingress of apparently inferior solutions in the optimal Pareto front by measuring the distance between the mean values of the fitness samples of two individuals with respect to the scaled average of their fitness variance. A probabilistic dominance approach has been adopted in [21] by Hughes to address the uncertainty of decision concerned with the superiority of a trial solution over another in noisy environment. In [34], a linear relationship between the sample size of a trial solution and the fitness variance in its local neighborhood is employed for non-uniform sampling. Moreover statistical expectation of fitness samples is used as the fitness measure of the trial solutions. Among the other popular works, the introduction of the principles of noise-aware α-dominance [3], noise tolerant dominance dependent lifetime [6], 'soft' selection [39], and surrogate model [22,23] need special mentioning. The paper extends the existing techniques of noisy MOO by the following counts.

1. Existing approaches of noisy MOO introduce periodic evaluation (sampling) of trial solutions to eliminate the risk of promoting poor solutions to the Pareto fronts. Traditional approaches employ fixed sample size, irrespective of the local noise variance of the trial solutions [3,40]. Sample size here is modeled as a monotonic non-decreasing function of the fitness variance in the neighborhood (capturing the local noise variance) of a trial solution with an aim to allow more (less) sample size for trial solutions with larger (smaller) local noise variance.
2. Traditional noisy evolutionary/swarm algorithms estimate fitness of a trial solution by taking the average of the corresponding fitness samples [26,30,31]. Unfortunately, the amplitude of noise linked up with different fitness samples vary apparently randomly, thereby introducing uncertainty in the estimation of the respective fitness. An *Interval Type-2 Fuzzy Set* based model has been employed here to handle the uncertainty in fitness estimation using noisy samples of a trial solution.
3. Classical MOO algorithms employ Pareto dominance conditions to judiciously place selective trial solutions in the Pareto front [7,12]. These conditions being deterministic cannot capture the right candidates, when the objectives are contaminated with noise. The present paper extends the Pareto dominance conditions by incorporating probabilistic estimate of dominance of a trial solution over other.
4. Because of noisy fitness estimates, sometimes a relatively good trial solution is discarded from being pulled up for the optimal Pareto front. Goldberg et al. in [2] proposed a statistical test to include even a slightly weaker solution in the optimal Pareto front. In this paper, we propose an alternative, more viable statistical test criterion, for inclusion of weaker trial solutions, having non-uniform sample size, in the optimal Pareto front.
5. The diversity of trial solutions of the selected lowest rank Pareto front is established in traditional MOO algorithms by introducing the concept of crowding distance [12,34]. This paper considers both crowding distance and probability of dominance while promoting trial solutions from the selected lowest rank Pareto front to the next generation to maintain both diversity and quality of solutions in presence of noise.

Experiments have been undertaken to examine the potency of the proposed noisy MOO algorithm realized with Differential Evolution for Multi-objective Optimization (DEMO) [35] (hereafter called *DEMO in presence of Noise with Stochastic selection*–DEMONS) by injecting noise samples (into the objective space) of three different types of distribution, including (i) Gaussian noise with mean zero and increasing variance; (ii) Poisson noise with increasing variance and (iii) random noise with positive and negative expeditions of the noise amplitude within $\pm 25\%$ of the true fitness function values. Experiments undertaken reveal that DEMONS outperforms its competitors [2,3,6,22,34,39] when used to optimize noisy versions of 23 recommended benchmark functions [46] with respect to *inverted generational distance*, *spacing*, *error ratio* and *hyper volume ratio*metrics.

The paper is divided into five sections. Section 2 provides an overview of Type-2 fuzzy sets, MOO and the DEMO algorithm. Section 3 provides the noise handling mechanism in DEMONS. Experimental settings for the benchmarks and the simulation strategies are explained in Section 4. Conclusions are given in Section 5.

2. Preliminaries

In this section, we provide an overview of three important concepts: Type-2 fuzzy sets, Multi-Objective Optimization (MOO), Differential Evolution for Multi-objective Optimization (DEMO), which will be used in the rest of the paper to develop a solution to the noisy MOO problems.