Information Sciences 305 (2015) 93–96

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A note on "Numerical solutions for linear system of first-order fuzzy differential equations with fuzzy constant coefficients"

Marzieh Najariyan [a], Mehran Mazandarani [b,*]

[a] Department of Applied Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran
[b] Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ARTICLE INFO

Article history:
Received 1 September 2014
Received in revised form 23 November 2014
Accepted 1 February 2015
Available online 7 February 2015

Keywords:
Fuzzy differential equations
Ordinary differential equations

ABSTRACT

In this note, it is shown that some obtained results in Solaymani Fard amd Ghal-Eh (2011) are not true.

1. Introduction and basic concepts

Throughout this note, the set of all real numbers is denoted by $\mathbb{R}$, and the set of all type-1 fuzzy numbers on $\mathbb{R}$ by E_1.The left and right end-points of α-*level* sets of $\widetilde{A}$, $\left[\widetilde{A}\right]^\alpha$, are denoted by $\underline{A}^\alpha$ and $\overline{A}^\alpha$ respectively.

The product of two type-1 fuzzy numbers $\tilde{u}$ and $\tilde{v}$, $\tilde{w} = \tilde{u}\tilde{v}$, in the α-*level* is as follows: $[\tilde{w}]^\alpha = [\underline{w}^\alpha, \bar{w}^\alpha] = [\tilde{u}\tilde{v}]^\alpha$, in which $\underline{w}^\alpha = \min\{\bar{u}^\alpha \underline{v}^\alpha, \bar{u}^\alpha \bar{v}^\alpha, \underline{u}^\alpha \bar{v}^\alpha, \underline{u}^\alpha \underline{v}^\alpha\}$ and $\bar{w}^\alpha = \max\{\bar{u}^\alpha \underline{v}^\alpha, \bar{u}^\alpha \bar{v}^\alpha, \underline{u}^\alpha \bar{v}^\alpha, \underline{u}^\alpha \underline{v}^\alpha\}$.

The triangular type-1 fuzzy number $\tilde{u} \in E_1$ is characterized by $\tilde{u} = (a, b, c)$ where $a \leqslant b \leqslant c$. The left and right end-points of the $\alpha - level$ sets of $\tilde{u}$ are determined as $b - (1-\alpha)(b-a)$ and $b + (1-\alpha)(c-b)$, respectively.

In [1], an iterative method has been proposed for solving the following class of Fuzzy Differential Equations [FDEs] system:

$$\dot{\widetilde{X}}(t) = \widetilde{A}\widetilde{X}(t) + \tilde{f}(t), \quad \widetilde{X}(0) = \widetilde{X}_0 \tag{1.1}$$

where $\dot{\widetilde{X}}(t) = \frac{d\widetilde{X}(t)}{dt}$, $\widetilde{X} : [0, t_f] \to E_1^n$ is a fuzzy function on the real interval $[0, t_f]$, $\widetilde{A} = [\tilde{a}_{ij}]_{n\times n}$ is a matrix, $\tilde{a}_{ij} \in E_1$, $\tilde{f} : [0, t_f] \to E_1^n$ is a fuzzy function, and $\widetilde{X}_0$ is the initial value of the system.

2. The note

In [1], FDEs system (1.1) has been transformed to the following two systems of Ordinary Differential Equations [ODEs]

* Corresponding author.
E-mail addresses: Marzieh.najariyan@gmail.com (M. Najariyan), me.mazandarani@gmail.com, me.mazandarani@ieee.org (M. Mazandarani).

http://dx.doi.org/10.1016/j.ins.2015.02.006
0020-0255/

$$\begin{cases} \underline{\dot{X}}^{\alpha}(t) = \underline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha} + \underline{f}^{\alpha}(t) & \underline{X}^{\alpha}(0) = \underline{X}_0^{\alpha} \\ \dot{\overline{X}}^{\alpha}(t) = \overline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha} + \bar{f}^{\alpha}(t) & \overline{X}^{\alpha}(0) = \overline{X}_0^{\alpha} \end{cases} \tag{2.1}$$

where $\widetilde{X}(t)$ is Hukuhara differentiable, i.e. $\left[\dot{\widetilde{X}}(t)\right]^{\alpha} = \left[\underline{\dot{X}}^{\alpha}(t), \dot{\overline{X}}^{\alpha}(t)\right]$; and $\underline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha}$, $\overline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha}$ are determined as:

$$\underline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha} = \min\left\{BU \middle| B \in \left[\underline{A}^{\alpha}, \overline{A}^{\alpha}\right], U \in \left[\underline{X}^{\alpha}(t), \overline{X}^{\alpha}(t)\right]\right\},$$
$$\overline{\left[\widetilde{A}\widetilde{X}(t)\right]}^{\alpha} = \max\left\{BU \middle| B \in \left[\underline{A}^{\alpha}, \overline{A}^{\alpha}\right], U \in \left[X^{\alpha}(t), \overline{X}^{\alpha}(t)\right]\right\}.$$

In [1], based on relationship (2.1), the following ODEs systems have been claimed to be the transformation of FDEs system (1.1) provided that $\overline{A}^{\alpha}$ is non-positive:

$$\begin{cases} \underline{\dot{X}}^{\alpha}(t) = \underline{A}^{\alpha}\overline{X}^{\alpha}(t) + \underline{f}^{\alpha}(t) & \underline{X}^{\alpha}(0) = \underline{X}_0^{\alpha} \\ \dot{\overline{X}}^{\alpha}(t) = \overline{A}^{\alpha}\underline{X}^{\alpha}(t) + \bar{f}^{\alpha}(t) & \overline{X}^{\alpha}(0) = \overline{X}_0^{\alpha} \end{cases} \tag{2.2}$$

The point is that, ODEs systems (2.2) cannot be considered, in general, as the transformation of FDEs system (1.1). As a matter of fact, the functions $\underline{X}^{\alpha}(t)$ and $\overline{X}^{\alpha}(t)$ are unknown, and they may be positive and/or negative on the real interval $t \in [0, t_f]$. To illustrate, consider the following one-dimensional FDE:

$$\dot{\tilde{x}}(t) = \tilde{k}\tilde{x}(t) \qquad \tilde{x}(0) = (-10, -9, -8) \qquad t \in [0, 5] \tag{2.3}$$

where $\tilde{k} = (-3, -2, -1)$. Based on the relationship (2.2), FDE (2.3) is transformed to

$$\begin{cases} \underline{\dot{x}}^{\alpha}(t) = (-2 - (1-\alpha))\bar{x}^{\alpha}(t) & \underline{x}^{\alpha}(0) = -9 - (1-\alpha) \\ \dot{\bar{x}}^{\alpha}(t) = (-2 + (1-\alpha))\underline{x}^{\alpha}(t) & \bar{x}^{\alpha}(0) = -9 + (1-\alpha) \end{cases} \tag{2.4}$$

which means

$$\underline{\left[\tilde{k}\tilde{x}(t)\right]}^{\alpha} = \min\{BU | B \in [-2 - (1-\alpha), -2 + (1-\alpha)], U \in [\underline{x}^{\alpha}(t), \bar{x}^{\alpha}(t)]\} = (-2 - (1-\alpha))\bar{x}^{\alpha}(t),$$

$$\overline{\left[\tilde{k}\tilde{x}(t)\right]}^{\alpha} = \max\{BU | B \in [-2 - (1-\alpha), -2 + (1-\alpha)], U \in [\underline{x}^{\alpha}(t), \bar{x}^{\alpha}(t)]\} = (-2 + (1-\alpha))\underline{x}^{\alpha}(t).$$

According to FDE (2.3), it is obvious that there exists $t' \in [0, 5]$ such that $\underline{x}^{\alpha}(t') < 0$, $\bar{x}^{\alpha}(t') < 0$. Then, it can be concluded that $(-2 + (1-\alpha))\underline{x}^{\alpha}(t') \geqslant (-2 + (1-\alpha))\bar{x}^{\alpha}(t') > 0$ and $(-2 - (1-\alpha))\underline{x}^{\alpha}(t') \geqslant (-2 - (1-\alpha))\bar{x}^{\alpha}(t') \geqslant (-2 + (1-\alpha))\bar{x}^{\alpha}(t') > 0$. Consequently, we have $\underline{\left[\tilde{k}\tilde{x}(t')\right]}^{\alpha} = (-2 + (1-\alpha))\bar{x}^{\alpha}(t')$ and $\overline{\left[\tilde{k}\tilde{x}(t')\right]}^{\alpha} = (-2 - (1-\alpha))\underline{x}^{\alpha}(t')$ which mean ODEs system (2.4) cannot be considered as the transformation of FDE (2.3).

As a result, based on the product of two type-1 fuzzy numbers, the ODEs (2.2) are true on condition that $\overline{A}^{\alpha}$ is non-positive, and $\underline{X}^{\alpha}(t)$ is non-negative for $\alpha \in [0, 1]$ and $t \in [0, t_f]$. It is noteworthy that, in [1], no condition is assumed on functions $\underline{X}^{\alpha}(t)$ and/or $\overline{X}^{\alpha}(t)$. Based on the aforementioned, the obtained results in example 5.2 for $t > 0.1$ are not true. As a way of

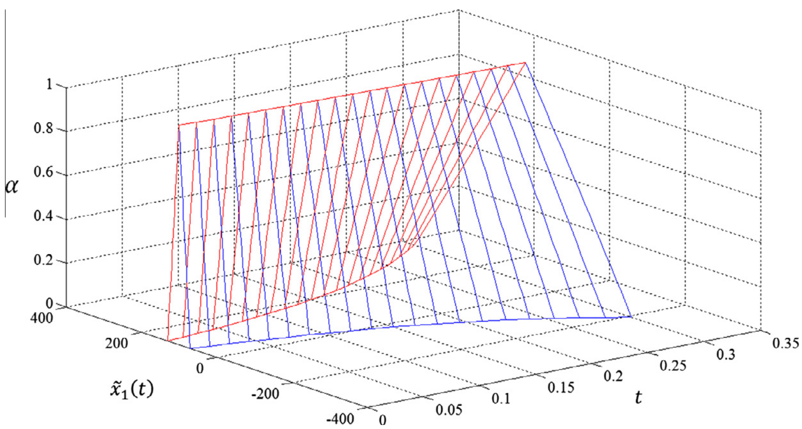

Fig. 1. The solution of example 5.2, $\tilde{x}_1(t)$, obtained in [1].