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Priority based ϵ dominance: A new measure in multiobjective optimization

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ABSTRACT

Dominance is a fundamental concept in Multiobjective Optimization (MOO) where a solution is said to dominate the other if it is better in at least one objective, and same as or better in the other objectives. Given a pair of solutions, if neither dominates the other, then the solutions are said to be non-dominated. In real spaces, a large number of non-dominated solutions can exist in principle, making convergence of MOO techniques rather slow. Relaxed forms of dominance have been proposed in the literature for faster convergence. One such form is the ϵ-dominance, which in principle, partitions the space into ϵ-sized grids. The value of ϵ is often user defined and the performance of ϵ-dominance based multiobjective evolutionary algorithms (ϵ-MOEAs) critically depends on the value of ϵ. In this article we propose a novel way of determining the value of ϵ, which is different for each objective function, based on the correlation between them. We call this approach Priority Based ϵ (PBE) as the method ranks the objectives according to their priority before calculating the ϵ values. PBE is incorporated in an archived simulated annealing based MOO technique called AMOSA. AMOSA has been earlier shown to outperform several well-known MOO techniques especially for many objective optimization. PBE based AMOSA, referred to as PBE-AMOSA, is found to comprehensively outperform AMOSA, MOEA/D-DE, the conventional ϵ based version ϵ-AMOSA and ϵ-MOEA both in case of benchmark test problems and 0/1 knapsack problem.

1. Introduction

In a multi-objective optimization (MOO) problem, a number of objectives are simultaneously minimized or maximized. A single solution that simultaneously optimizes each objective does not exist for a nontrivial multi-objective optimization problem. In this case, the objective functions are said to be conflicting, and so there exists possibly infinite number of Pareto optimal solutions. These solutions are non-dominated to each other so it cannot be stated conclusively that one solution is better than other. Multiple conflicting objectives are natural in most real world scenarios. Several metaheuristics, including those based on evolutionary algorithms have been suggested over the last decade [7,9] for solving MOO problems. Recently a simulated annealing based multi-objective (MO) algorithm, proposed by Bandyopadhyay et al. [3], is found to provide improved performance as compared to some well known Multi-Objective optimization algorithms, namely NSGA-II [1], PAES

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[30], MOSA [46], for a large number of test problems, especially for cases with many objectives. The primary goal of an MOO strategy is to provide *non-dominated* solutions as output while maintaining the following two properties.

- **Convergence**: *The algorithm should quickly converge to true Pareto-Optimal set (i.e., true solutions).*
- **Diversity**: *The algorithm should maintain a spread of solutions.*

In many of the proposed MO algorithms the convergence property is primarily taken into consideration while less emphasis is given on the diversity property of the solutions. Deb [9] suggested a steady state MOEA which tries to maintain diversity in the solutions while attempting to converge towards the true Pareto-Optimal front. However there is no proof for its convergence property. In all of the previous MOO algorithms the following aspects are not adequately addressed:

- Good convergence while maintaining diversity among solutions
- Rapid increase of the number of non-dominated solutions with the increase in number of objectives in the finite search spaces. It is not possible for the decision maker to select a solution from such a large set.

In the following subsection, we discuss some of the popular algorithms developed in order to address the above problems.

1.1. Related works

Several researchers have proposed alternative form of dominance relation. Farina and Amato [16] introduce a new kind of fuzzy dominance, namely $(1-k)$ dominance by modifying the definition of conventional Pareto-dominance. The dominance relation tries to address the problem of impractical size of the solution set for many-objective problems (i.e., problems with higher number of objectives). Several alternative dominance relations like preference order ranking [14] and relation controlling with dominance area [43] had been proposed in the past few years. Laumanns et al. [32] propose another kind of dominance relation, known as ϵ-dominance, which guarantees the convergence towards the true Pareto front while maintaining diversity among the solutions. The value of ϵ is either an input from the user or computed from the number of solutions required. A very large value of ϵ is useful to speed up the algorithm while sacrificing the goodness of the solutions obtained. In contrast, if good solutions are needed (solutions "near" to the Pareto set), one should go for the small value of ϵ which leads to slower convergence rate. Therefore, it is important to choose an appropriate value of the ϵ. ϵ-dominance basically tries to incorporate some amount of relaxation (which depends on the ϵ value) in the dominance relation. The amount of relaxation can vary for each objective. In order to model a generalized dominance scheme which automatically determines suitable ϵ values for different objectives, Hernández-Díaz et al. [18] proposed a technique based on the geometric shape of the Pareto front. However, the method makes several assumptions regarding the nature of the front which makes it inapplicable for MOO problems with arbitrary Pareto fronts and higher number of objectives.

Several other works has also been proposed in the past few years. One of the main task of the decision maker is the use of reference points and achievement scalarizing functions. The main idea of these approaches is to convert the problem into a single objective problem using a scalarizing function based on a reference point. But Deb [13] suggests that a single point is less effective than an area around the single reference point. Molina et al. [37] proposed a new kind of dominance relation, **g-dominance** which using the knowledge of the reference point can estimate an area around the reference point. This method does not demand any scalarizing function and can be incorporated in any MO algorithm. Several other approaches using scalarizing or indicator functions had been suggested [28,19–21,24,23,47,34]. In many objective problem, the number of objectives have a direct influence on the hardness (can be convergence rate toward the Pareto Optimal set) of the problem. In many of the past works, the number of nondominated solutions had been considered in order to deal with the evolution hardness. One such remedy is to restrict the number of non dominated solutions. Several other researchers try to reduce the number of objectives [12,5,26,27]. Saxena and Deb [12] proposed a principal component analysis based objective reduction technique. Jaimes et al. [26] used correlation between objectives to reduce the number of objectives. Schütze et al. [45] proposed an alternative way that focuses towards the ability to evolve towards the Pareto set. They have incorporated a descent cones (adapted from [6]) to measure the probability to improve a solution by generational operators. In order to visualize the non dominated solutions in many objective framework several visualization techniques [40,48,31,36] had been proposed that maps the high dimensional objectives into lower dimension. Several other researchers have proposed new ways of contribution in the domain of optimization [17,25,8,42].

A new type of MOO algorithm has been proposed by Zhang and Li [33,49]. Their method MOEA/D decomposes the multi-objective into several single objective sub problems. Then using population-based methods they optimize them simultaneously. Their method is a generic framework and does not require any pareto-dominance concept. An extensive survey of multiobjective optimization algorithms has been well addressed in [38,39].

Recently, Bandyopadhyay et al. [4] has incorporated the concept of ϵ-dominance in AMOSA, termed as ϵ-AMOSA. As mentioned in [32], selection of proper ϵ value is a crucial step. As depending on ϵ value, the "quality" of the solution set is affected. In our knowledge, there is one work [18], in which authors addressed and solved this problem. But, the major restrction of their work is that they assumed the optimization problem to be bi- or tri-objective. This is a serious limitation as in industrial applications, there are lot of problems where number of objective is in the order of 10, 20. And as we studied the