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The approximation number function and the characterization of covering approximation space

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ABSTRACT

There is renewed interest in the topic of covering-based rough sets. It is important to understand the characterization of covering approximation spaces. Recently, Zhu et al. presented the concept of an approximation number function, which can be viewed as a quantitative tool for analyzing the covering approximation spaces. We focus on this concept and show that an approximation number function can be characterized by its properties. We give the axiomatic definition of an approximation number function, and prove that there is a one-to-one correspondence between the set of all lower approximation number functions and the set of all covering approximation spaces. This suggests that the concept is fundamental to the characterization of covering approximation spaces. We apply approximation number functions to the existing studies on approximation spaces, and a series of matroidal structures are induced in an approximation space.

1. Introduction

Rough set theory is a useful tool to manage the vagueness, granularity, and uncertainty in information systems. In 1983, Zakowski [29] proposed the concept of covering-based rough set approximations. A pair of lower and upper approximation operators were defined by generalization of the Pawlak definition. Covering-based rough sets have subsequently attracted more research interest [1,2,27,30,31]. A review of existing studies shows a great diversity of research. Different covering models were established [3,7,21,22,32,35], minimal covering reducts were defined [8,13,14,26,31], axiomatic systems were built [9,23,31], and the relationships among rough set models were investigated [19,27,28]. They were applied to knowledge reduction [5,15,36], feature selection algorithm design [4,10], and so forth.

Previous studies on covering-based rough sets are extremely rich, which provides a solid foundation for combining them with other theories. Recently, the integration of covering-based rough sets and matroids attracted increased research interest [11,20,24,25,33,34]. A matroid is a mathematical structure providing a unifying abstract treatment for graph theory, linear algebra, and combinatorial optimization. Matroids have been used in many fields, but particularly in the field of greedy algorithm design. Many important problems in rough set theory, including attribute reduction, are non-deterministic polynomial-time hard (NP-hard), and therefore the algorithms for solving them are usually greedy [34]. Therefore, it would be advantageous to combine matroids with rough sets to solve optimization problems. This study focuses on the relationship between matroids and rough sets [11,20,24,25,33]. To establish a relationship between generalized rough sets and matroid

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theory, Zhu and Wang [25,33] proposed the concept of an approximation number function for covering-based approximation spaces. This concept can be viewed as a quantitative tool for analyzing covering approximation spaces. We consider the relationship between covering approximation spaces and this new concept, and whether the concept allows further research on the combination of matroid theory and rough sets.

Similarly to the rank function of a matroid, Zhu and Wang [25,33] defined an approximation number function for covering-based approximation spaces, establishing a relation between generalized rough sets and matroid theory. The rank function is a fundamental concept in matroid theory, and a matroid can be defined from the viewpoint of rank functions. We investigated if an approximation number function plays a similar role to a rank function. There were two main research directions. First, we investigated if an approximation number function was an important quantitative tool in approximation spaces, similarly to the rank function. Second, we determined if an approximation space can be defined from the viewpoint of approximation number functions, again similarly to rank functions. The first aspect has been investigated previously [25]. However, we take the discussion somewhat further and establish a connection between an upper approximation number function (UAF) and a lower approximation number function (LAF). Hence the propositions of a UAF can be inferred from an LAF, and vice versa. We focused on the investigation of an LAF. We show that an approximation number function can be characterized by its properties, and the axiomatic definition of an approximation number function is given. There is a one-to-one correspondence between the set of all LAFs and the set of all covering approximation spaces, which addresses the second topic. Using the concept of approximation number functions, we derive the relationship between them, and show that a series of matroidal structures can be induced in approximation spaces.

Section 2 reviews some fundamental concepts regarding rough sets and matroids. Section 3 proposes some new properties of an LAF and shows that an LAF can be characterized by three axioms. Section 4 discusses the application of approximation number functions in the studies on approximation spaces. Some connections between approximation spaces and matroids is established in terms of approximation number functions. Concluding remarks are given in Section 5.

2. Basic definitions

We recall some fundamental definitions related to covering-based rough sets and matroids.

For a set U the family of all subsets of U is $P(U) = \{X|X \subseteq U\}$. For any $X \in P(U)$, the complement of X is $\sim X = U - X$, and the cardinality of set X is $|X|$.

2.1. Covering-based rough sets

Let U be a finite non-empty set of objects and $\mathcal{C}$ a family of subsets of U. If no subset in $\mathcal{C}$ is empty then $\cup\mathcal{C} = U$, where $\mathcal{C}$ is a covering of U.

Definition 1 (*Approximation space [18,29]*). Let U be a finite non-empty set of objects and $\mathcal{C}$ be a covering of U. Then the ordered pair $(U, \mathcal{C})$ is called an approximation space.

In rough sets, a pair of approximation operators are used to describe an object. For a covering $\mathcal{C}$ of U, the principal lower approximations for $X \subseteq U$ are

$$\underline{\mathcal{C}}(X) = \cup\{C \in \mathcal{C}|C \subseteq X\} \text{ and } \underline{\mathcal{C}}(X) = \cup\{N(x)|x \in U \wedge N(x) \subseteq X\}.$$

The latter can be seen as the particular case of the former [19]. The lower approximation, $\underline{\mathcal{C}}(X) = \cup\{C \in \mathcal{C}|C \subseteq X\}$, is widely used in the covering rough set model. Following Ref. [25,33], the concept of UAFs is defined by means of the upper approximation, $\overline{\mathcal{C}}(X) = \cup\{C \in \mathcal{C}|C \cap X \neq \emptyset\}$.

Definition 2 (*Approximation operators [18,29]*). Let $(U, \mathcal{C})$ be an approximation space. For $X \subseteq U$, the lower approximation operator and the upper approximation operator of $(U, \mathcal{C})$ are defined as follows:

$$\underline{\mathcal{C}}(X) = \cup\{C \in \mathcal{C}|C \subseteq X\}; \quad \overline{\mathcal{C}}(X) = \cup\{C \in \mathcal{C}|C \cap X \neq \emptyset\}.$$

Definition 3 (*Upper approximation number function [25,33]*). Let $(U, \mathcal{C})$ be an approximation space. For all $X \subseteq U$,

$$UN_{\mathcal{C}}(X) = |\{C \in \mathcal{C}|C \cap X \neq \emptyset\}|$$

is called the upper approximation number of X, and $UN_{\mathcal{C}}$ is called a UAF with respect to $\mathcal{C}$.

The upper approximation number of a covering approximation space is similar to the dimension of a vector space or the rank of a matrix. Therefore, we may consider a UAF to be an upper approximation rank. However, we retain the term approximation number function to be consistent with Ref. [25,33].

An LAF can be similarly defined.

Definition 4 (*Lower approximation number function*). Let $(U, \mathcal{C})$ be an approximation space. For all $X \subseteq U$,