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Optimal linear estimation with square-based sampling

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ABSTRACT

This paper is concerned with the problem of event-based sampling for the purpose of reducing the number of data transferred in control systems and the corresponding modified Kalman filter. A mathematical description based upon square-based sampling is proposed, in which the sampling occured when the accumulated difference between two adjacent sampling points satisfies the specified event condition. Then, by utilizing the square based sampling method, a modified Kalman filter algorithm is adopted, in which the measurement is sent when the output exceeds a given event. The simulation results illustrate the advantage of the new event-based sampling scheme and the effectiveness of modified Kalman filter.

1. Introduction

Recently, with the rapid development of digital measurements and intelligent instruments, digital signal processing methods have almost replaced the commonly adopted analog signal processing methods. They provide better reliability, accuracy and stable performance. In many practical situations, the output signal of the plant is measured at sampled points [2]. The principle of signal processing maintains that a signal must be sampled at a Nyquist rate i.e. at least twice its bandwidth in order to be represented without error. And most of the current implementations of digital control and estimation use regular sampling with fixed period T [1], which means that the sampling is driven only by time and is also called Riemann sampling [29]. The feature of this approach is that the analysis and design become very simple. But in order to get the exact model, we should have as many samples as we can to describe the signal features. Clearly, this is wastage of valuable sensing/sampling resources. To save computer power and resources, varying sampling intervals may be used.

There are several alternatives to periodic time triggered sampling. The irregular observations are defined to be message-based observations with possibly infrequent and non-periodic measurements. That means that samples will be taken “when something interesting” happens [3] or when the signal passes some prespecified level. The feature of this approach is that sampling does not happen unless it is required. The aim is to reduce the sampling frequency and to achieve the same performance as achieved with periodic sampling. This type of sampling can be called Lebesgue sampling [16], or event-based sampling [11,31]. Up to now, many event-based sampling schems have been proposed, such as send-on-delta sampling [14,27], level-crossing sampling [7,9], deadband sampling [20], send-on-area sampling [18], and error energy sampling [13,15]. Although these schemes have different definitions, the common attribute is that the signal is sampled only when something interesting happens. The event-based sampling has been used widely in networked control systems [6,19,23–25], manufacturing systems [8,21,22], and communication systems [17,12].

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In this paper, a new event-based sampling, square-based sampling (SBS), has been discussed, in which the event using the difference of square between the adjacent sampling time for a general type continuous-time signal and the square is called the threshold. The SBS can get the exact time intervals of the sampling because it can detect the small changes between the adjacent sampling points. It also can accumulate the disturbance and overcome the difficulty that the sampling may not be triggered during a long time since the signal changes are not large enough to activate the level or threshold in the level-crossing or send-on-data sampling. It has been shown that under certain conditions the SBS can have a better performance than the periodic sampling, because the signal may not only be changed in small time intervals but also changed in time intervals with large disturbances.

State estimation problem has received considerable attention in the fields of control and signal processing over the past decades, because it is very important for the application perspective point of view in manufacturing, process control, mechatronics, biomedical systems, and many more. One of the most sophisticated state estimation algorithms is the well known Kalman filtering [4,26], which has been applied in many research fields. In this paper, we propose a modified Kalman filter based on the square-based sampling, in which fewer measurement samples are required to achieve a similar performance. Samples are not generated synchronously in time but only when a priori defined threshold occurs. More precisely, when a sample happens the estimated state is updated using the sampling measurement, while at synchronous time instants the update is based on the sampling knowledge. Otherwise the previous sample is used until the next sampling happens.

Throughout the paper, R^n denotes the n-dimensional Euclidean space; The notation $X > Y$ ($X \geqslant Y$) means that the matrix $X - Y$ is positive definite ($X - Y$ is semi-positive definite, respectively); I is the identity matrix of appropriate dimensions; For any matrix A, A^T denotes the transpose of matrix A, A^{-1} denotes the inverse of matrix A.

2. Event-based sampling

2.1. System model

Consider a continuous-time linear time invariant system given by:

$$\dot{x}(t) = Ax(t) + w(t) \tag{1}$$

$$y(t) = Cx(t) + v(t) \tag{2}$$

where $x(t) \in R^n$ is the state of the plant, $y(t) \in R^m$ is the measurement output which is sent to the estimator node by the sensor node. $w(t) \in R^n$ is the process noise, and $v(t) \in R^m$ is the measurement noise. It is assumed that $w(t)$ and $v(t)$ are uncorrelated, zero mean white Gaussian random processes, satisfying:

$$E\{w(t)w(s)'\} = Q_w \delta(t-s) \tag{3}$$

$$E\{v(t)v(s)'\} = Q_v^{old} \delta(t-s) \tag{4}$$

$$E\{w_i(t)v_j(s)\} = 0, \quad 1 \leqslant i \leqslant n, \quad 1 \leqslant j \leqslant q \tag{5}$$

2.2. Square-based sampling

The square-based sampling scheme is illustrated in Fig. 1(a), where t_i, t_{i-1} are the adjacent sampling times, $y(t_i)$, $y(t_{i-1})$ are the corresponding sampling values, the square $S > 0$ as the "event" is the difference accumulation between the two adjacent sampling points and is also called threshold. Compare with the send-on-delta sampling (illustrated in Fig. 1(b). The signal will be sampled only when the value changes more than the specified Δ value. The sampling may not be triggered during a long time since the signal changes are not large enough to trigger the next sampling), we can see that the SBS can detect the

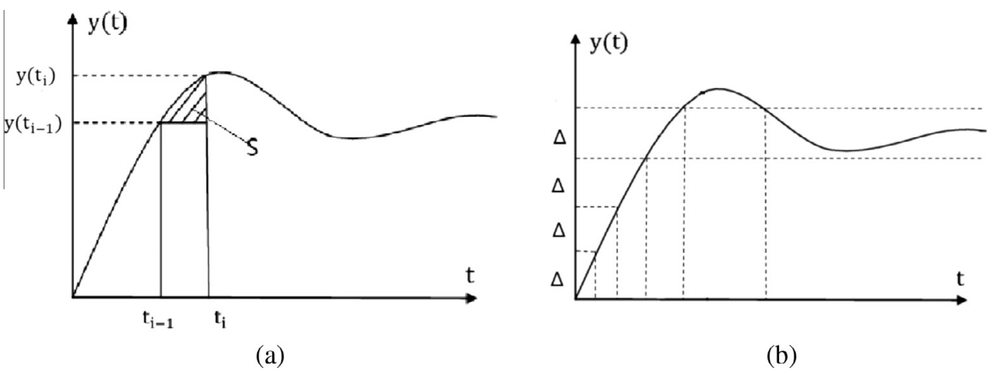

Fig. 1. (a) Principle of square-based sampling. (b) Principle of send-on-delta sampling.