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Dual-sparsity regularized sparse representation for single image super-resolution

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ABSTRACT

Recently, by exploring the column nonlocal similarity prior among the sparse representation coefficients, the column nonlocal similarity sparse representation models for solving the ill-posed single image super-resolution (SISR) problem are attracting more and more attention. However, these conventional models consider only the prior among nonlocal similar sparse representation coefficients, and fail to consider the prior among all entries (or rows) of the sparse representation coefficient. Hence the modeling capability may be limited. In fact, if a cluster of similar representation coefficients is rearranged into a matrix in the sparse representation coefficient space, the nonlocal similarity priors exist both among columns and rows. Using the row nonlocal similarity prior, a row nonlocal similarity regularization term with l_1-norm constraint is explored. By introducing it to the conventional column nonlocal similarity sparse representation model, we present a dual-sparsity regularized sparse representation (DSRSR) model. A surrogate function based iterative shrinkage algorithm is introduced to effectively solve the proposed model. Extensive experiments on SISR demonstrate that the presented model can effectively reconstruct the edge structures and suppress the noise, achieving convincing improvement over many state-of-the-art example-based methods in terms of PSNR, SSIM and visual quality.

1. Introduction

Super-resolution (SR) is a very active research topic in the image processing community. Many tasks, such as remote sensing, medical diagnostic and consumer electronics, rely on high quality images for reliable and accurate analysis as well as prediction. However, in many practical situations, due to the inherent limitations of the optical system or other factors, the observed images are often of low resolution (LR), thus limiting the subsequent tasks based on them [24]. Image SR aims to reconstruct a high resolution (HR) image from a single or a set of LR observations, which is termed as single image SR and multiimage SR, respectively [35].

The focus of this paper is single image super-resolution (SISR). For an observed image y, the problem of SISR can be generally formulated by [6,33,11,35,9,10]

$$y = DHx + v, \tag{1}$$

where x and y are lexicographically stacked representations of the original image and the observed image, respectively. H is a blurring matrix and D is a downsampling matrix. v is a noise vector.

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To cope with the ill-posed nature of the SISR problem, the regularization-based techniques have been widely used to regularize the solution space [6,23,3,38]. In [6], The classic iterative back-projection technique was proposed to reconstructs x by minimizing $\hat{x} = \arg\min_x \left\{ \|y - DHx\|_2^2 \right\}$. However, the solution to this l_2-norm optimization is generally not unique, and hence the reconstructed image tends to produce visual artifacts, such as ringing, aliasing and blocking artifacts. To further refine the solution space, some effective regularization terms of image x, denoted by $R(x)$, can be introduced to regularize the solution: $\hat{x} = \arg\min_x \left\{ \|y - DHx\|_2^2 + \lambda \cdot R(x) \right\}$, where λ is a Lagrangian multiplier parameter, which balances the tradeoff between the regularization term $R(x)$ and likelihood term $\|y - DHx\|_2^2$. One widely used regularization term is total variation (TV) [23], which assumes that natural images have small first derivatives. Mathematically, it is set as $R(x) = |\nabla x|_1$, where $|\nabla x|_1$ is the l_1-norm of the first-order derivative of x. But, it favors piecewise constant image structures, and hence tends to smooth much the image details. To improve the classic TV regularization term, other regularization terms, such as the adaptive TV regularization term [3] and the nonlocal TV regularization term [28], have also been developed.

Another class of the SISR technique is the example-based techniques [12,2,15,18,33,14,35,17,31,34,36,4,7,26,37], which assume that high-frequency details lost in a single LR image can be predicted from a training data set. In terms of the example-based technique, the sparse representation techniques [33,11,21,9,10,20,19,25] have attracted more and more attention in recent years. These techniques assume that the image is sparse in some domain spanned by a set of bases or a dictionary of atoms. Here the sparsity prior means that the image x can be well approximated by a linear combination of selected atoms from an over-complete dictionary matrix Ψ, i.e., $x \approx \Psi s$, and most of the entries in the coefficient vector s are zero or close to zero. Mathematically, the sparsity regularization term can be set as $R(s) = \|s\|_0$, where $\| \|_0$ is a pseudo norm that counts the number of non-zero entries in s. Since l_0-norm optimization is NP-hard problem [29], the l_1-norm sparsity regularization term as the closest convex relaxation, $R(s) = \|s\|_1$, is widely adopted [33,11,9,10]. It leads to the following Lagrangian form: $s_y = \arg\min_s \{\|y - DH\Psi s\|_2^2 + \lambda\|s\|_1\}$, where λ is a constant controlling the sparsity and the approximation error. The l_1-norm optimization problem can be solved by techniques such as the surrogate function based iterative shrinkage algorithm [8] and proximal algorithms [5]. Given a single LR image y, the coefficient s_y can be solved with the l_1-norm sparsity regularization term and the dictionary Ψ, and then the desired HR image can be reconstructed by $\hat{x} = \Psi s_y$. Clearly, it is expected that the coefficient s_y could be close enough to the true coefficient of the original image x. Due to the degradation of the observed image, however, it is a challenge to obtain the true coefficient from the degraded image y. Using only the local l_1-norm sparsity regularization term $R(s) = \|s\|_1$ may not lead to an enough accurate image SR [10]. Recently, the nonlocal similarity prior among the sparse representation coefficients (or called as the column nonlocal similarity prior) as another prior has been introduced into sparse representation model for better image SR performance [11,21,9,10]. This prior assumes that the coefficient s can be well approximated by its nonlocal similar coefficients in the sparse representation coefficient space. Mathematically, the column nonlocal similarity regularization term can be defined as $R(s) = \|s - \beta\|_p$, where β is denoted as the weighted average of nonlocal similar coefficients associated with the coefficient s, p is typically chosen as 1 or 2 [11,21,9,10].

Although the column nonlocal similarity sparse representation models consider the relationship among the sparse representation coefficients, they fail to consider the relationship among all entries (or rows) of the sparse representation coefficient. Hence the modeling capability may be limited. In [39], the authors stated that if a cluster of similar image patches is rearranged to form a matrix in image patch space, the nonlocal similarity priors exist both among columns and rows. In [21,9], the authors stated that the nonlocal similar image patches have similar sparse representation coefficients. Namely, if a cluster of similar representation coefficients is rearranged into a matrix in the sparse representation coefficient space, the nonlocal similarity priors exist both among columns and rows as well.

In this paper, we present a dual-sparsity regularized sparse representation (DSRSR) model. First, using the row nonlocal similarity prior, a row nonlocal similarity regularization term with l_1-norm constraint is explored. Second, this regularization term is introduced to the conventional column nonlocal similarity sparse representation model to form the DSRSR model. The surrogate function based iterative shrinkage algorithm [8] is employed to effectively solve the presented DSRSR model. Extensive experiments on SISR demonstrate that the presented model can effectively reconstruct the edge structures and suppress the noise, achieving convincing improvement over many state-of-the-art example-based methods in terms of Peak Signal to Noise Ratio (PSNR) [30], Structural Similarity (SSIM) [13] and visual quality assessment.

The rest of the paper is organized as follows: Section 2 describes the details of the presented DSRSR model for SISR. Section 3 provides the iterative shrinkage algorithm for solving the DSRSR model. Section 4 presents the extensive experimental results together with relevant discussions and Section 5 concludes the paper.

2. The presented DSRSR model

For the ease of description, we introduce some denotations that will be used in the following content. Let $x \in R^N$ be an image vector, and $x_i = P_i x$ be the ith patch (size: $\sqrt{n} \times \sqrt{n}$) vector of x, where $P_i \in R^{n \times N}$ is a matrix extracting patch x_i from x. For patch x_i, suppose that a dictionary $\Psi \in R^{n \times m}$ is selected for it. Then, x_i can be approximated as $x_i \approx \Psi s_{x,i}$ by $s_{x,i} = \arg\min_s \left\{ \|x_i - \Psi s_i\|_2^2 + \lambda\|s_i\|_1 \right\}$, where $s_{x,i} \in R^m$. The whole image x can be reconstructed by averaging all of the reconstructed patches x_i. Mathematically, it can be written as [11,10]