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A geometric and game-theoretic study of the conjunction of possibility measures

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ABSTRACT

In this paper, we study the conjunction of possibility measures when they are interpreted as coherent upper probabilities, that is, as upper bounds for some set of probability measures. We identify conditions under which the minimum of two possibility measures remains a possibility measure. We provide graphical way to check these conditions, by means of a zero-sum game formulation of the problem. This also gives us a nice way to adjust the initial possibility measures so their minimum is guaranteed to be a possibility measure. Finally, we identify conditions under which the minimum of two possibility measures is a coherent upper probability, or in other words, conditions under which the minimum of two possibility measures is an exact upper bound for the intersection of the credal sets of those two possibility measures.

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1. Introduction

1.1. Possibility Measures: Why (Not)

Imprecise probability models [36] are useful in situations where there is insufficient information to identify a single probability distribution. Many different kinds of imprecise probability models have been studied in the literature [37]. It has been argued that closed convex sets of probability measures, also called *credal sets*, provide a unifying framework for many—if not most—of these models [36,24].

A downside of using credal sets in their full generality is that they can be computationally quite demanding, particularly in situations that involve many random variables. Therefore, in practice, it is often desirable to work with simpler models whose practicality compensate their limited expressiveness. *Possibility measures* [39,15,8,10] are among the simplest of such models, and present a number of distinct advantages:

- Possibility measures can be easily elicited from experts, either through linguistic assessments [9] or through lower confidence bounds over nested sets [29].
- Possibility distributions provide compact and easily interpretable graphical representations.
- In large models, when exact computations are costly, possibility measures can be simulated straightforwardly through random sets [1] (for example to propagate uncertainty through complex models [2]).

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- Lower and upper expectations induced by possibility measures can be computed exactly by Choquet integration [32, Section 7.8].
- When interpreted as sets of probability measures, possibility measures have a limited number of extreme points [25,22]. Many inference algorithms, for instance many of those used in graphical models, employ extreme point representations: using possibility measures in such algorithms will reduce the computational effort required.

An obvious disadvantage of using a family of simpler models is that the family may not be rich enough to allow certain standard operations. For instance, multivariate joint models obtained from possibilistic marginals are usually not possibility distributions [26], hence outer-approximating possibility measures have been proposed [33,13] to allow one to use the practical advantages of such models.

1.2. Formulation of the problem

In this paper, we focus exclusively on the *conjunction* of two models, that is, the intersection of two credal sets. The conjunction is of interest, for instance, when possibility measures have been elicited from different experts, and we want to know which probability measures are compatible with the assessments of all experts simultaneously. As such, the conjunction is a combination rule that aggregates pieces of information consisting of several inputs to the same problem.

Many combination rules for imprecise probability models are discussed in the literature; see for instance [6,27,18,20,4,11]. In this paper, we define the conjunction of two possibility measures as the upper envelope of the set of probability measures that are compatible (i.e., dominated) by both. The following questions arise:

- It may happen that there is no probability measure that is compatible with both possibility measures, in which case the conjunction does not exist. In the language of imprecise probability theory, this means that the conjunction *incurs sure loss*. When does this happen?
- Even when there is at least one probability measure that is compatible with both possibility measures, the upper envelope may not be a possibility measure. In order words, it is not guaranteed that the conjunction on possibility measures is *closed* [18]. When is the conjunction of two possibility measures again a possibity measure? If it is not, can we effectively approximate it by a possibility measure?
- Finally, if the conjunction is a possibility measure, can we express that possibility measure directly in terms of the two possibility measures that we are starting from, without going through their credal sets?

We will answer each of the questions above, using the notions of avoiding sure loss, coherence and natural extensions from the behavioural theory of imprecise probabilities [36]. The main contributions of this paper are:

- From a theoretical viewpoint, we provide sufficient and necessary conditions for the intersection to be again a credal set that can be represented by a possibility measure (Theorems 14 and 16).
- From a practical perspective, we derive from these conditions correction strategies such that the intersection of the corrected models is an outer-approximating possibility distribution (Lemma 21 and Theorem 22).

Interestingly, some of our results can be proven quite elegantly by means of standard results from zero-sum game theory (Theorem 15). This theory also leads us to a graphical method to check the conditions and to apply the correction strategy (Section 4.3).

1.3. Related literature

The literature on the conjunction of possibility measures is somewhat scarce. However, there are quite a few related results that have been proven in the context of evidence theory, which from a formal point of view includes possibility theory as a particular case.

The compatibility of two possibility measures, meaning that the intersection of their associated sets of probabilities is non-empty, was characterized by Dubois and Prade in [17]. Related work for belief functions was done by Chateauneuf in [5].

With respect to the conjunction of two possibility measures again being a possibility measure, a necessary condition is the *coherence* of the minimum of these two possibility measures. This coherence was investigated by Zaffalon and Miranda in [40]. We are not aware of any necessary and sufficient conditions for the conjunction determining a possibility measures, and the only existing results are counterexamples showing that this need not be the case: see [17], and also [5] for the case of belief functions.

A related problem that has received more attention is the connection between conjunction operators of possibility theory and the conjunction operators of evidence theory: for example Dubois and Prade [16] study how Dempster's rule relate to possibilistic conjunctive operators, and Destercke and Dubois [12] relate belief function combinations to the minimum rule of possibility theory.

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