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Fuzzy and interval-valued fuzzy decision-theoretic rough set approaches based on fuzzy probability measure

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ABSTRACT

This paper investigates decision-theoretic rough set (DTRS) approach in the frameworks of fuzzy and interval-valued fuzzy (IVF) probabilistic approximation spaces, respectively. It takes fuzzy probability and IVF probability into consideration. Bayesian decision procedure is a basis of DTRS approach. By integrating fuzzy probability measure and IVF probability measure into Bayesian decision procedure, there come fuzzy decision-theoretic rough set (FDTRS) approach and interval-valued fuzzy decision-theoretic rough set (IVF-DTRS) approach. The new approaches have the ability to directly deal with real-valued and interval-valued data. This makes FDTRS and IVF-DTRS more applicable than DTRS. Two methods are presented to compare intervals while constructing the IVF-DTRS approach: one is compatible with DTRS and FDTRS approaches; the other is a total order based on which the decision procedure is much easier to operate. Cases of two different universes of discourse for FDTRS and IVF-DTRS are also taken into account.

1. Introduction

The decision-theoretic rough set (DTRS) approach was proposed by Yao et al. [39,45,46] as a kind of generalization of rough set approach [29,30]. With the aid of Bayesian decision procedure, DTRS approach offers a mathematical way to interpret thresholds in probabilistic rough set (PRS) [14,40,55,56]. This approach is fulfilled by splitting the approximated set into three regions, positive, boundary and negative regions, guided by the idea of three-way decisions. Three-way decisions [8,41–43], just as the name, consist of three different kinds of rules—positive rules (corresponding to positive region), boundary rules (corresponding to boundary region) and negative rules (corresponding to negative region). It is an important application of rough set and granular computing [31]. The idea of three-way decisions is superior to that of the traditional two-way decisions, which only bring about positive rules and negative rules (to make decisions of acceptance and rejection, respectively). The boundary rules, in three-way decisions, are to make decisions of deferment. The DTRS approach based on Bayesian decision procedure has been demonstrated to be useful in text classification [13,16], web-based support systems [44], cluster analysis [15,22,49], investment decisions [25], multi-classification [21,3], email filtering [11,53] and government decisions [23], etc.

In DTRS approach conditional probability and loss function play an important role in determining thresholds according to which decisions are made. Many researchers have discussed different types of loss function. Li defined a general loss function for supervised learning in which the cost and benefit of assigning an instance to a specific subcategory was evaluated [13].

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Different decision makers may hold different attitudes towards a same action. Then, Li and Zhou gave three assumptions for the values of losses and proposed a three-way view decision model in which optimistic, pessimistic and equable decisions are provided [17]. Later, Li et al. proposed cost-sensitive classification based on DTRS model [18]. They assumed that the total cost in the classification should include a misclassification cost and a test cost, and designed an algorithm for searching an optimal test attributes set with minimum cost. Lingras et al. defined various loss functions for different objects in the context of rough clusters and proposed a cluster quality index based on DTRS model [22]. Considering the imprecision embodied within uncertainty, Liang and Liu studied decision-theoretic rough set with triangular fuzzy loss function and interval-valued loss function in [20,19], respectively, and proposed triangular fuzzy decision-theoretic rough set (TFDTRS) and interval-valued decision-theoretic rough set (IVDTRS). In the framework of shadowed sets [31], Deng and Yao proposed the decision-theoretic three-way approximations of fuzzy sets [4]. Their main idea is to find out the required pair of thresholds to minimize the over-all cost of three-way approximations.

In spite of all these developments on loss function, there are few researches on studying the conditional probability. Among many literatures [10,17,19,24,26,32,33,39–43,53,54], the conditional probability in applications is estimated by membership functions. That is, let X be a classical subset and R be a classical equivalence relation. Then the conditional probability in these literatures is obtained by $P(X|[x]_R) = \frac{|X\cap[x]_R|}{|[x]_R|}$, in which $|\cdot|$ is the cardinality of a classical set. This computation method is based on the assumption of uniform probability distribution of the universe. There exist other methods to estimate conditional probability. Yu et al. calculated the possibility of two objects assigned into one cluster depending on the similarity between them [49]. However, this calculation is based on similarity not on real probability. In [23,48], the Bayes' theorem was adopted to infer the conditional probability. But it is difficult to get those joint probabilities, especially when the objects are not independent. The fuzzy probabilistic rough set based on two universes has been discussed by Yang et al. [38]. Even though they have considered fuzzy relation in their model, it is the λ-cut sets of fuzzy relation instead of the fuzzy relation itself that works when computing the conditional probability. Also, it is difficult to choose a reasonable λ. Sun et al. studied the probabilistic rough fuzzy sets in the framework of probabilistic approximation space [34]. Their model does take fuzzy set into consideration but still through classical equivalence relations. The aforementioned methods show disadvantages for computing conditional probability. Besides, real-valued data and interval-valued data make them more inconvenient to function. For example, when dealing with a fuzzy information system, the fuzzy relation obtained from data should be first converted into classical relation in case of computing probability, just like in [38]. This is inconvenient and, sometimes, causes information loss for improper λ. Thus, for the convenience of directly dealing with fuzzy and IVF information, we need to generalize the DTRS approach based on fuzzy and IVF probabilities, which are the main works of this paper.

The remainder of this paper is organized as follows. Section 2 reviews basic notions of fuzzy set, fuzzy probability measure, IVF set, etc. A new method of division of interval numbers is introduced. Section 3 considers DTRS approach with fuzzy probability based on one and two universes, separately. Thus, FDTRS and generalized FDTRS are obtained, respectively. Several illustrative examples are presented together. Section 4 studies IVF-DTRS approach which are based on interval-valued loss function and IVF probability. Two different methods of comparing intervals are provided. Examples are given to explain our model clearly. The last section concludes this paper.

2. Preliminaries

In order to make this paper self-contained, we recall some useful concepts of fuzzy sets, fuzzy probability measure, IVF sets, etc. More details can be found in [1,50,51].

2.1. Fuzzy set and fuzzy probability

Let U be a universe of discourse. A fuzzy set A is a mapping from U into $[0,1]$, i.e. $A : U \to [0,1]$. The family of all fuzzy sets on U is denoted by $\mathscr{F}(U)$. The product and complement of fuzzy sets from $\mathscr{F}(U)$ are defined, respectively, as follows: for all $x \in U$,

$$(AB)(x) = A(x)B(x),\ A^c(x) = 1 - A(x).$$

Let $A, B \in \mathscr{F}(U)$. Then, $A \subseteq B \Longleftrightarrow A(x) \leqslant B(x), \forall x \in U$.

The α-cut sets of fuzzy sets take an important role in connecting with classical set theory. For $A \in \mathscr{F}(U)$ and $\alpha \in [0,1]$, the set $A_\alpha = x \in U : A(x) \geqslant \alpha$ is called the α-cut set of A.

Let U and V be two universes of discourse and R be a mapping from $U \times V$ to $[0,1]$. Then R is a fuzzy relation from U to V. If $U = V$, then R is a fuzzy relation on U, i.e. $R \in \mathscr{F}(U \times U)$. Furthermore, if $R(x,x) = 1$ for each $x \in U$, then R is reflexive; if $R(x,y) = R(y,x)$ for all $x, y \in U$, then R is symmetric; if $R(x,y) \geqslant \vee_{z\in U}(R(x,z) \wedge R(z,y))$ for all $x, y \in U$, then R is transitive.

We present the definition and properties of fuzzy events as follows according to [51].

Definition 2.1 [51]. Let $(\Omega, \mathscr{A}, P)$ be a probability space. If

$$A \in \zeta(\mathscr{A}) = \{A \in \mathscr{F}(\Omega) : A_\alpha \in \mathscr{A}, \alpha \in [0,1]\},$$

then A is a fuzzy event on Ω. The probability of A is