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On performance evaluation for a multistate network under spare routing

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ABSTRACT

This paper focuses on a multistate network composed of multistate edges to study the relationship between transmission reliability and spare routing. In the network, each edge has several possible capacities and may fail due to failure, maintenance, etc. Hence, the minimum transmission time to send a given amount of data is not a fixed number. The spare routing is a transmission rule which indicates the first and the second priority pairs of minimal paths. The second one takes charge of the transmission duty if the first one is out of order. We evaluate the probability that the required amount of data can be sent through a pair of minimal paths simultaneously under both time threshold and budget constraint. Such a probability is named transmission reliability which can be regarded as a performance index to measure the transmission capability of a multistate network. An efficient solution procedure is thus proposed to generate all lower boundary points meeting the constraints. The transmission reliability is calculated in terms of such points.

1. Introduction

The shortest path problem, which finds a path with minimum length, is one of the well-known and practical problems in operations research, computer science, networking, and other areas. When data/commodities are transmitted through a flow network, it is desirable to adopt the least cost path, largest capacity path, shortest delay path, or some combination of multiple criteria [1,4,12,13], which are all variants of the shortest path problem. From the point of view of quality management and decision making, it is an important task to reduce the transmission time through the network. Hence, a version of the shortest path problem called the quickest path problem is proposed by Chen and Chin [7]. This problem finds a path with minimum transmission time to send a given amount of data through the network. Such a path is named the quickest path. In this problem, each edge has the capacity and the lead time contributes [7,16,27,28]. More specifically, the capacity and the lead time are both assumed to be deterministic. Several variants of quickest path problems are thereafter proposed; constrained quickest path problem [6,10], the first k quickest paths problem [8,9,11,29], and all-pairs quickest path problem [5,18].

However, due to failure, partial failure, maintenance, etc., each edge is multistate in many real-life flow networks such as computer, telecommunication, urban traffic, and logistics systems. That is, each edge has multiple capacities or states [17,19–26,34,35]. Such a network is named a multistate network. For instance, a computer system with each edge representing the transmission medium and each node representing the station of servers is a typical multistate network. Virtually, each physical line such as coaxial cables and fiber optics has only normal or failure state. Each transmission medium consisting of several physical lines has several states in which state k means k physical lines are operational. Therefore, the minimum transmission time to send a given amount of data through a multistate network is not fixed.

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From the viewpoints of decision making and Quality of Service (QoS) [14,30,32], it is a critical issue to reduce the transmission time for a multistate network. Hence, the data can be transmitted through a pair of disjoint minimal paths (MPs) simultaneously, where a MP is a path without loops. Besides, cost is another crucial factor in enterprise competing. The budget constraint is thus included in the addressed problem. We mainly evaluate the probability that the multistate network can send d units of data through a pair of MPs under both time threshold T and budget B. Such a probability is named transmission reliability, which can be regarded as a performance index to measure the transmission capability of a multistate network. Furthermore, the network administrator decides the spare routing in advance to indicate the first and the second priority pairs of MPs. The second one will be responsible for the transmission duty if the first one is out of order. A simple algorithm is first proposed to generate all lower boundary points for (d, T, B, Q_j), the minimal capacity vectors sending d units of data through Q_j under T and B. The notation Q_j denotes jth pair of MPs, $j = 1, 2, \ldots, g$, where g is the number of pairs of MPs. The transmission reliability can then be calculated in terms of such points by applying inclusion–exclusion. The remainder of this work is organized as follows. The network model and the algorithm to generate all lower boundary points for (d, T, B, Q_j) are both proposed in Section 2. Both the spare routing and the corresponding transmission reliability are presented in Section 3. An example is demonstrated in Section 4 to illustrate the algorithm and how the transmission reliability may be calculated. Computational time complexity of the proposed algorithm is analyzed in Section 5. The simplified network model and more discussion about the spare routing are presented in Section 6.

2. Problem formulation and the algorithm

Let $G \equiv (N, E, L, M, C)$ denote a multistate network with a source s and a sink t where N denotes the set of nodes, $E \equiv \{e_i | 1 \leqslant i \leqslant n\}$ denotes the set of edges, $L \equiv (l_1, l_2, \ldots, l_n)$ with l_i denoting the lead time of e_i, $M \equiv (M_1, M_2, \ldots, M_n)$ with M_i denoting the maximal capacity of e_i, and $C \equiv \{c_i | 1 \leqslant i \leqslant n\}$ with c_i denoting the transmission cost on e_i. The capacity of edge e_i, denoted by x_i, takes possible values $0 = b_{i1} < b_{i2} < \cdots < b_{ir_i} = M_i$, where b_{ij} is an integer for $j = 1, 2, \ldots, r_i$. The vector $X \equiv (x_1, x_2, \ldots, x_n)$ is called the capacity vector of G. Such a G is assumed to further satisfy the following assumptions:

1. Each node is perfectly reliable.
2. Each edge has multiple states with a given probability distribution.
3. The capacities of different edges are statistically independent.
4. The data are transmitted through two disjoint MPs simultaneously.

Definition 1. $Y \geqslant X$ means that $y_i \geqslant x_i$ for each $i = 1, 2, \ldots, n$ where $Y = (y_1, y_2, \ldots, y_n)$ and $X = (x_1, x_2, \ldots, x_n)$.

Definition 2. $Y > X$ means that $Y \geqslant X$ and $y_i > x_i$ for at least one i.

2.1. Transmission time

Suppose $P_1, P_2, \ldots, P_m$ are MPs of G. If d units of data are transmitted through the MP $P_a = \{e_{a1}, e_{a2}, \ldots, e_{an_a}\}$, $a = 1, 2, \ldots, m$, then the total cost $Z(d, P_a)$ is

$$Z(d, P_a) = \sum_{k=1}^{n_a} (d \cdot c_{ak}), \tag{1}$$

where $(d \cdot c_{ak})$ is the cost through e_{ak} for $k = 1, 2, \ldots, n_a$. Note that not all MPs or all Q_j are utilized during the evaluation process of transmission reliability. The pair Q_j consists of two MPs; P_{j1} and P_{j2}. The following equation states that the total cost through both P_{j1} and P_{j2} cannot exceed the budget,

$$Z(d_{j1}, P_{j1}) + Z(d_{j2}, P_{j2}) \leqslant B, \tag{2}$$

where d_{j1} and d_{j2} are the assigned demand through P_{j1} and P_{j2}, respectively. For convenience, let $\lambda_j = \{(d_{j1}, d_{j2}) | (d_{j1}, d_{j2})$ satisfies the budget constraint$\}$. Then the capacity of MP P_a under X is $\min_{1 \leqslant k \leqslant n_a}(x_{ak})$, $a = 1, 2, \ldots, m$. Hence, the transmission time [7] to send d units of data through P_a under X, $\chi(d, X, P_a)$, is

$$\text{lead time of } P_a + \left\lceil \frac{d}{\text{the capacity of } P_a} \right\rceil = \sum_{k=1}^{n_a} l_{ak} + \left\lceil \frac{d}{\min\limits_{1 \leqslant k \leqslant n_a} x_{ak}} \right\rceil, \tag{3}$$

where $\lceil x \rceil$ is the smallest integer such that $\lceil x \rceil \geqslant x$. It contradicts the time threshold if $\chi(d, X, P_a) > T$. We have the result of Lemma 1.

Lemma 1. *$\chi(d, X, P_a) \geqslant \chi(d, Y, P_a)$ if $X < Y$ where Y is a capacity vector.*

Proof. If $X < Y$, then $x_{ak} \leqslant y_{ak}$ for each $e_{ak} \in P_a$, and $\min_{1 \leqslant k \leqslant n_a} x_{ak} \leqslant \min_{1 \leqslant k \leqslant n_a} y_{ak}$. Thus, $\left\lceil \frac{d}{\min_{1 \leqslant k \leqslant n_a} x_{ak}} \right\rceil \geqslant \left\lceil \frac{d}{\min_{1 \leqslant k \leqslant n_a} y_{ak}} \right\rceil$. We complete the proof by obtaining that $T \geqslant \chi(d, X, P_a) \geqslant \chi(d, Y, P_a)$. □