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# Generalised operations on hesitant fuzzy values in the framework of Dempster–Shafer theory

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ABSTRACT

The hesitant fuzzy sets theory (*HFS*) is probably the latest generalisation of fuzzy sets theory and seems to be especially useful in the solution of multiple criteria group decision making (*MCGDM*) problems, where it enables us to avoid some specific problems concerned with the aggregation of expert's opinions. Currently there are only few different definitions and generalisations of *HFS* proposed in the literature. The key issue of hesitant fuzzy sets theory is the formulation of operation laws on the hesitant fuzzy elements (*HFE*), as they make it possible to use *HFS* for the solution of real-world problems. This paper presents a critical analysis of conventional operations on *HFE* and their applicability to the solution of multiple criteria decision making (*MCDM*) problems. It is shown that the known approaches to the definitions of *HFS* and corresponding operation laws have some important limitations and drawbacks. Therefore, a new generalised definition of *HFS* and operation laws based on the interpretation of intuitionistic fuzzy sets in the framework of the Dempster–Shafer theory of evidence (*DST*) are proposed and analysed. With the use of corresponding theorems it is proved that the proposed approach is free of limitations and drawbacks of known methods.

The corresponding methods for aggregation of local criteria presented by *HFE*s in the framework of *DST* are proposed and analysed. The proposed approach allows us to solve *MCDM* and *MCGDM* problems without intermediate defuzzification when not only criteria, but their weights are *HFE*s. The advantages of the proposed approach are illustrated with numerical examples and the case study.



## 1. Introduction

Torra and Narukawa [29] and Torra [28] proposed the hesitant fuzzy set, which permits the membership to have a set of possible values, and discussed the relationship between the hesitant fuzzy set and intuitionistic fuzzy set. They showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set, but only a few works devoted mainly to the solution of *MCGDM* problems in the hesitant fuzzy setting were published in the literature.

Hesitant fuzzy sets seem to be the latest generalisation of fuzzy sets. To make our analysis more transparent we consider here an informal introduction to the problem presented in [42] as follows.

Suppose we have three groups of experts which proposed a degree to which an alternative satisfies a criterion. Some experts propose 0.3, some provide 0.5, and the others propose 0.6. These three groups of experts insist on their estimations.

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Therefore the degree that the alternative should satisfy the criterion can be represented by a hesitant fuzzy set $\{0.3, 0.5, 0.6\}$. It is important that the hesitant fuzzy set $\{0.3, 0.5, 0.6\}$ describes the above situation more objectively than the interval-valued fuzzy set [0.3,0.6] since the degrees that the alternative should satisfy the criterion are not the convex of 0.3 and 0.6, or the interval between 0.3 and 0.6, but just three possible values.

The most fruitful application of hesitant fuzzy sets is the solution of *MCGDM* problems.

Therefore Xia and Xu [41] developed a set of aggregation operators for hesitant fuzzy information and applied them to the solution of *MCDM* problem. Later, some induced aggregation operators in hesitant fuzzy setting are introduced by Xia et al. [36]. Based on quasi arithmetic means, Xia et al. [37] discussed some ordered aggregation operators and induced ordered aggregation operators, as well as their application in the group decision making. In [17,47], the concepts of *VIKOR* and *TOPSIS* methods are extended to develop a methodology for solving *MCDM* problems with hesitant fuzzy information.

In [2], the authors proposed a new approach, named HF-ELECTRE II approach that combines the idea of Hesitant Fuzzy Sets with the ELECTRE II method for the solution of *MCDM* problems.

Wei [34] developed some prioritized aggregation operators for aggregating hesitant fuzzy information, and then applied them to develop some models for hesitant fuzzy *MCDM* problems in which the attributes are in different priority level. The authors of the paper [50] explore the geometric Bonferroni mean (*GBM*) (which can capture the interrelationships among arguments and plays a crucial role in *MCDM*) considering both the Bonferroni mean and the geometric mean under hesitant fuzzy environment. In [23], the authors extended hesitant fuzzy sets by intuitionistic fuzzy sets and refer to them as generalised hesitant fuzzy sets. They redefined some basic operations on generalised hesitant fuzzy sets. A generalised hesitant fuzzy synergetic weighted distance (*GHFSWD*) measure, which is based on the generalised hesitant fuzzy weighted distance (*GHFWD*) measure and the generalised hesitant fuzzy ordered weighted distance (*GHFOWD*) measure proposed by Xu and Xia [40] is presented in [22] with application to the solution of *MGCDM* problems. In [35], the authors considered the multiple attribute decision making problems in which attribute values take the form of hesitant interval-valued fuzzy information. The definition and some operation laws on hesitant interval-valued fuzzy elements and score function of hesitant interval-valued fuzzy elements were introduced and some hesitant interval-valued fuzzy aggregation operators were developed. A new type of fuzzy preference structure, called interval-valued hesitant preference relations, is proposed in [4] to describe uncertain evaluation information in *MGCDM* processes. In [48], the author developed a wide range of hesitant fuzzy power aggregation operators for hesitant fuzzy information. Some similarity measures and correlation measures on the hesitant fuzzy sets were investigated by Xu and Xia [40,41]. The authors of paper [3] proposed some correlation coefficient formulas for hesitant fuzzy sets and applied them to clustering analysis under hesitant fuzzy environments. Some information measures for hesitant fuzzy sets and interval-valued hesitant fuzzy sets were proposed in [12]. The correlation coefficient of dual hesitant fuzzy sets and its application to *MCDM* were proposed and analysed in [46]. The methods for multi-attribute decision analysis under a linguistic hesitant fuzzy environment were developed in the papers [6,14,16,18–20,32,33]. Induced generalised hesitant fuzzy operators and their application to multiple attribute group decision making are presented in [49]. The state of art in the field of hesitant fuzzy sets and possible future directions are presented in the recent paper [24].

It should be emphasised that in all above cited papers, the operation laws defined on hesitant fuzzy sets and elements in [28,29] were used.

It is very important to note that Torra and Narukawa [29] and Torra [28] proved that the operations on hesitant fuzzy sets and hesitant fuzzy elements which they proposed are consistent with those of intuitionistic fuzzy sets (*IFS*) when applied to the envelope of hesitant fuzzy set.

Therefore, the operation laws of *IFS* are currently used for the solution of different problems in the framework of *HFS* [3,23,34,35,47,48,50].

Nevertheless, it was shown in the papers [11,39,43] that classical operations on intuitionistic fuzzy values (*IFV*) have some undesirable properties and may provide counterintuitive results.

Therefore, no wonder that some operations on hesitant fuzzy elements defined in [28,29] have the same undesirable properties and may provide wrong solutions of *MCGDM* problems (we shall show this in Section 3).

On the other hand, in [10,11], we showed that there is strong link between the intuitionistic fuzzy sets theory and the Dempster–Shafer theory of evidence (*DST*). In [11], we showed that the operations on intuitionistic fuzzy values may be rewritten in terms of *DST* and substituted with the operations on belief intervals. With the use of corresponding theorems, it was proved in [11] that a new set of operations is free of drawbacks of the classical operations on intuitionistic fuzzy values. Moreover, the semantic of *DST* makes it possible to introduce new operations which were not defined in terms of intuitionistic fuzzy sets theory.

Therefore, in the current paper, we shall show that a belief interval may be treated as an envelope of hesitant fuzzy elements and redefine operations on hesitant fuzzy elements in terms of *DST*. We shall show that these operations are free of drawbacks of those defined in [28,29] and introduce a new useful operation not defined in [28,29].

For these reasons, the rest of this paper is set out as follows.

In Section 2, we recall briefly the basics of intuitionistic fuzzy sets theory and *DST*, and present the set of operations on belief intervals which are free of drawbacks of classical operations on intuitionistic fuzzy values. Section 3 is devoted to a new definition of hesitant fuzzy elements and its possible generalisation. The numerical examples illustrating the applicability of the proposed method for the solution of *MCDM* problems are presented as well. The practical example illustrating new possibilities of the proposed approach is presented in Section 4. Finally, the concluding section summarises the paper.