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An optimization-based approach to adjusting unbalanced linguistic preference relations to obtain a required consistency level

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ABSTRACT

In certain real decision-making situations, decision makers often feel more comfortable providing their knowledge and preferences in linguistic terms. Unbalanced linguistic term sets may be used in decision problems with preference relations. However, the lack of consistency in decision-making with linguistic preference relations can lead to inconsistent conclusions. Based on the consistency measure of unbalanced linguistic preference relations, this paper proposes an optimization-based approach to improving the consistency level of unbalanced linguistic preference relations. This consistency-improving model preserves the utmost original knowledge and preferences in the process of improving consistency. Furthermore, it guarantees that the elements in the optimal adjusted unbalanced linguistic preference relation are all simple unbalanced linguistic terms. Finally, we propose a mixed 0–1 linear programming aimed to obtain the optimum solution to the proposed consistency improving model and to demonstrate its practicability.

1. Introduction

Preference relations are popular techniques used to model decision makers' knowledge and preferences regarding decision problems. There are two varieties of preference relations: numerical preference relations [7,14,20,22,31,33,37] and linguistic preference relations [10,24,35]. The consistency issue is a very important problem in decision-making with preference relations. Since the lack of consistency may lead to inconsistent conclusions [15,20,23,38].

In some real decision-making situations, decision makers often feel more comfortable providing their knowledge and preferences linguistically. Solving a decision problem with linguistic information implies the need for computing with words [16,24,25,28]. In particular, Herrera and Martínez [17] proposed the 2-tuple linguistic representation model to deal with uniformly and symmetrically distributed linguistic term sets. There has been much research on the question of consistency measures (e.g., [2,3,6,9,34]) regarding 2-tuple linguistic preference relations. Alonso et al. [2,3] and Cabrerizo et al. [6] proposed an interesting consistency index based on additive transitivity and linguistic 2-tuples. Dong et al. [9] developed a 2-tuple linguistic index to measure the consistency degree of linguistic preference relations. The Dong et al. [9] index not only measures the consistency degree of linguistic preference relations linguistically, but also reflects individual differences in the consistency degree of linguistic preference relations.

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However, the 2-tuple linguistic representation model only guarantees accuracy when dealing with a uniformly and symmetrically distributed linguistic term set. In recent years, the methodologies based on linguistic 2-tuples have been developed to deal with term sets that are non-uniformly and non-symmetrically distributed [1,4,11–13,19,21,26,32]. Herrera et al. [19] developed an interesting model called unbalanced linguistic term sets to handle such term sets. Nonetheless, there is little research on the question of consistency issues of unbalanced linguistic preference relations. Cabrerizo et al. [5] proposed the additive transitivity for unbalanced linguistic preference relations and presented a consistency index to measure the degree of consistency of unbalanced linguistic preference relations. However, Cabrerizo et al. [5] did not discuss how to improve the consistency level.

In this study, we discuss the consistency of unbalanced linguistic preference relations with the aim of obtaining a modified unbalanced linguistic preference relation with a required consistency level. To undertake this, we propose an optimization-based approach to obtaining an optimum solution. This optimization-based model preserves the utmost original preference information in the process of improving the consistency, according to the required consistency level. Moreover, it guarantees that the elements of the optimal adjusted unbalanced linguistic preference relations are all presented in simple unbalanced linguistic terms. Meanwhile, we demonstrate that a mixed 0–1 linear programming can be used to obtain the optimum solution to this optimization-based model.

The remainder of this paper is organized as follows: Section 2 introduces the 2-tuple linguistic methodology to deal with unbalanced linguistic term sets [19], as well as the consistency measure of unbalanced linguistic preference relations presented in Cabrerizo et al. [5]. This is followed by Section 3 that presents an optimization-based approach to improving the consistency level in unbalanced linguistic preference relations. Section 4 then provides an illustrative example. Section 5 then concludes this paper with final remarks.

2. Preliminaries

This section introduces the 2-tuple linguistic methodology dealing with unbalanced linguistic term sets [19] and presents the consistency measure of unbalanced linguistic preference relations in Cabrerizo et al. [5] that provides the basis for this study.

2.1. Linguistic methodology to deal with unbalanced linguistic term sets

The basic notations and operational laws of linguistic variables are introduced in [19,25,27,30,36]. Let $S = \{s_i | i = 0, 1, \ldots, g\}$ be a linguistic term set with odd cardinality. The term s_i represents a possible value for a linguistic variable and it is required that the linguistic term set is ordered: $s_i > s_j$ if and only if $i > j$.

The 2-tuple linguistic representation model, presented in Herrera and Martínez [17], represents the linguistic information by a 2-tuple (s_i,α), where $s_i \in S$ and $\alpha \in [-0.5,0.5)$. This linguistic model defines a function with the purpose of making transformations between linguistic 2-tuples and numerical values. Formally, let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0,g]$ a value representing the result of a symbolic aggregation operation. Then the 2-tuple that expresses the equivalent information to β is obtained by means of the following function:

$$\Delta : [0,g] \to S \times [-0.5, 0.5) \tag{1}$$

where

$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases}. \tag{2}$$

Clearly, Δ is a one to one mapping function. For convenience, its range is denoted as $\overline{S}$. Accordingly, Δ has an inverse function with $\Delta^{-1} : \overline{S} \to [0,g]$ and $\Delta^{-1}(s_i,\alpha) = i + \alpha$. In this study, (s_i, α) is denoted a simple term if $\alpha = 0$.

Unbalanced linguistic term sets have been proposed in [19]. Generally, an unbalanced linguistic term set S has a minimum term, a maximum term and a central term, and the remaining terms are non-uniformly and non-symmetrically distributed around the central one on both left and right lateral sets, i.e.,

$$S = S_L \cup S_C \cup S_R \tag{3}$$

where the left lateral set S_L contains all the terms smaller than the central term, where the central set S_C contains only the central term, and where the right lateral set S_R contains all the terms higher than the central term.

The concept of linguistic hierarchies [8,18], i.e., $LH = \cup_t l(t,n(t))$, is used to obtain 2-tuple linguistic representations of unbalanced linguistic values without a loss of information. A linguistic hierarchy is a set of levels in which each level is a linguistic term set with different granularity from the remaining levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as $l(t,n(t))$, with t being a number that indicates the level of the hierarchy and s_5 the granularity of the linguistic term set of t. Generally, the linguistic term set $S^{n(t+1)}$ of level $t+1$ is obtained from its predecessor $S^{n(t)}$ as $l(t,n(t)) \to l(t+1, 2, \ldots, n(t)-1)$. In linguistic hierarchies LH, the transformation function between terms from different levels to represent 2-tuple linguistic representations is as follows [18]: for any linguistic levels t and $t', TF^t_{t'} : l(t,n(t)) \to l(t',n(t'))$, so that