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# A novel approach to fault detection for fuzzy stochastic systems with nonhomogeneous processes

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ABSTRACT

In this paper, we consider a class of fuzzy stochastic systems with nonhomogeneous jump processes. Our focus is on the design of a fuzzy fault detection filter that is sensitive to faults but robust against unknown inputs. Furthermore, the error filtering system is stochastically stable. With reference to an $H_\infty$ performance index and a new performance index, sufficient conditions to ensure the existence of a fuzzy robust fault detection filter are derived. Simulation studies are carried out, showing that the proposed fuzzy robust FD filter can rapidly detect the faults correctly.



## 1. Introduction

In modern manufacturing systems, their operating conditions are highly complex [11]. Also, it is well known that when some components are malfunction, it will cause poor performance so much so that the system may become unstable. To improve safety and reliability of the manufacturing system, fault detection become an active research topic in the past decade. On the other hand, since the introduction of Takagi–Sugeno (T–S) fuzzy model [18], where a complex nonlinear system can be described in terms of a family of IF–THEN rules, the T–S fuzzy model based approach has been applied to the study of control problems involving nonlinear systems [5,4]. It includes studies on fault detection for T–S fuzzy-based nonlinear systems, (see, e.g., [14] and the references therein). However, the obtained results are for nonlinear systems with constant parameters, that is, these results are obtained under the assumption that there are no sudden switches nor stochastic disturbances. Clearly, this assumption is not realistic for many practical nonlinear systems. In reality, random abrupt changes or variations in structures or parameters are normal. They are caused by sudden environmental changes, stochastic switchings of subsystems and system noises. This is a major motivation for the investigation of Markov jump systems (MJSs), where the systems are T–S fuzzy based.

For Markov jump systems (MJSs), it has been an active research area since the publication of the pioneering work in [8]. The main reasons are: (i) MJSs can provide better models for practical systems with variations in parameters or structure, caused by sudden changes in environment, or operation conditions and (ii) The dynamical behaviors of MJSs can capture the phenomenon that occur in practical systems in areas, such as aerospace industry, manufacturing systems, economic

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systems, robotic manipulators [6], teleoperators [10], wheeled mobile manipulators [7] and electrical systems [12,2]. Issues relating to stability, stabilization, control and filtering for MJSs have been extensively studied (see, e.g., [24,3,17,16,20,21] and the references therein). It is worth mentioning that systems subject to Markov jump parameters may increase the possibility of failures, caused by undetected fault in one of the subsystems, which is crucial to the overall system.

In recent years, much work has been done on fault detection [27,25,26], under the assumption that the MJSs evolve as a homogeneous Markov process or Markov chain, (i.e. the transition probabilities of these systems are time-invariant), some works on FD for nonlinear MJSs have been reported in the literature (see, e.g., [22,23] and the references therein). However, this assumption is not realistic in many situations, such as the one in networked systems [9,15], where packet dropouts and network delays are different in different period, and so the transition rates are uncertain varying through the whole working region. Therefore, their transition probabilities are time-varying. Another example is a helicopter system [13], where the airspeed variation in such system are modeled as homogeneous Markov chain. However, their transition probabilities are not fixed due to changes in weather. Similar phenomenon is also observed in many other practical systems. In such situations, it is reasonable to model these systems by Markov jump systems with nonhomogeneous jump processes (chains) (i.e., the transition probabilities are time varying). A potential approach to deal with MJSs with nonhomogeneous jump processes (chains) is to use a polytope set to enclose the uncertainties caused by time-varying transition probabilities. For the transition probability of a Markov process which is not known exactly, it is possible to evaluate and hence obtain useful information in some working points. In this way, these time-varying transition probabilities can be modeled using a polytope, which is a convex set. This is the approach that is to be used in this paper to study fuzzy based nonlinear MJSs with time-varying transition probabilities.

So far, most of the results on control for fuzzy systems are obtained using one Lyapunov function. Therefore, these results tend to be conservative in nature, because they are obtained based on one common energy function being applied to all the linear sub-systems. To overcome the deficiency, a time-varying convex Lyapunov function is used in this paper to the system under consideration. Our focus is on the design of a fuzzy robust fault detection filter for a class of uncertain fuzzy-based nonlinear MJSs with nonhomogeneous jump processes. The system under consideration is subject to time-varying norm bounded parameter uncertainties. To begin, a robust fuzzy fault detection system and a filter-based residual generator are constructed. Then, an $H_\infty$ filtering system is designed so as to increase the robustness against unknown disturbances and parametric uncertainties. To continue, a new less conservative index is introduced, aiming to enhance the sensitivity to faults of the fuzzy fault detection system obtained. The fuzzy fault detection system is now having two conflicting performance induces. Thus, the fault detection problem is cast as an optimization problem, where an optimal trade-off point between robustness and sensitivity is to be obtained. For this, sufficient conditions expressed in terms of LMIs are derived based on which the desired fuzzy FD filter is constructed. Simulation studies are carried out so as to illustrate the effectiveness of the approach developed.

The rest of the paper is organized as follows: Section 2 contains problem statement and preliminaries results. In Section 3, stochastic stability analysis and threshold computation are given. In Section 4, $H_\infty$ performance and a new less conservative index for the resulting error dynamic system are analyzed, and a robust fuzzy fault detection filter is designed. A numerical example is given to illustrate the effectiveness of our approach in Section 5. Finally, some concluding remarks are given in Section 6.

In the sequel, the notation $\mathbb{R}^n$ stands for an $n$-dimensional Euclidean space, the transpose of a matrix $A$ is denoted by $A^{\mathrm{T}}; E\{\cdot\}$ denotes the mathematical statistical expectation; $L_2^n[0,\infty)$ stands for the space of $n$-dimensional square integrable functions over $[0,\infty)$; a positive-definite matrix is denoted by $P > 0; I$ is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

## 2. Problem statement and preliminaries

Let $(M, F, P)$ be a probability space, where $M, F$ and $P$ represent, respectively, the sample space, the algebra of events, and the probability measure defined on $F$. Let $\{r_k, k \geqslant 0\}$ be a discrete-time Markov stochastic process, which takes values in a finite state set $\Lambda = \{1, 2, 3, \ldots, N\}$, and $r_0$ represents the initial mode. The transition probability matrix is defined as

$$\Pi(k) = \{\pi_{ij}(k)\}$$

where $i, j \in \Lambda$, and $\pi_{ij}(k) = P(r_{k+1} = j | r_k = i)$ is the transition probability from mode $i$ at time $k$ to mode $j$ at time $k+1$, which satisfies $\pi_{ij}(k) \geqslant 0$ and $\sum_{j=1}^{N} \pi_{ij}(k) = 1$.

Consider an uncertain discrete-time nonlinear MJS with time-varying transition probability over the space $(M, F, P)$. We assume that it is represented by the following fuzzy model:

Plant rule $m$
IF $\theta_{1k}$ is $M_{m1}, \ldots$, and $\theta_{gk}$ is $M_{mg}$
THEN

$$\begin{cases} x_{k+1} = A_m(r_k)x_k + B_{fm}(r_k)f_k + B_{wm}(r_k)w_k + g_m(x_k, r_k) \\ y_k = C_m(r_k)x_k + D_{fm}(r_k)f_k + D_{wm}(r_k)w_k \end{cases} \tag{2.1}$$