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Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems

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ABSTRACT

An interval-valued hesitant fuzzy linguistic set (IVHFLS) can serve as an extension of both a linguistic term set and an interval-valued hesitant fuzzy set. This new set combines quantitative evaluation with qualitative evaluation; these can describe the real preferences of decision-makers and reflect their uncertainty, hesitancy, and inconsistency. This work focuses on multi-criteria decision-making (MCDM) problems in which the criteria are in different priority levels and the criteria values take the form of interval-valued hesitant fuzzy linguistic numbers (IVHFLNs). The new approach to solving these problems is based on the prioritized aggregation operators of IVHFLNs. Having reviewed the relevant literature, we provide interval-valued hesitant fuzzy linguistic operations and apply some linguistic scale functions, which have been improved on the basis of psychological theory and prospect theory. Ultimately, two kinds of prioritized aggregation operators of IVHFLNs are developed, which extend to a grouping prioritized situation and are applied to MCDM problems. Finally, an example is provided to illustrate and verify the proposed approach in two separate situations, which are then compared to other representative methods.

1. Introduction

Since the fuzzy set was proposed by Zadeh in 1965 [75], it has been widely researched and developed as well as being successfully applied in various fields [4,31,32,34,36,70,76]. In many multi-criteria decision-making (MCDM) problems, due to their fuzziness and uncertainty, the criteria weights and values of alternatives can be inaccurate, uncertain, or incomplete. Under these circumstances, fuzzy sets can provide robust solutions. In fuzzy sets, the membership degree of the element in a universe is a single value between zero and one; however, these single values are inadequate to provide complete information due to a lack of systematic and comprehensive knowledge.

Hesitant fuzzy sets (HFSs), an extension of traditional fuzzy sets, can address these situations. HFSs were first introduced by Torra and Narukawa [45,46], and they permit the membership degrees of an element to be a set of several possible values between zero and one. HFSs are highly useful in resolving situations where people hesitate when providing their preferences in the decision-making process, and they have been a subject of great interest to researchers. Recently, Rodríguez et al. [43] presented an overview and discussed future trends for HFSs. Farhadinia [12] proposed a series of score functions for HFSs. Wei [57], Zhang [81], Yu [74], and Ai et al. [1] studied the aggregation operators of HFSs. Farhadinia [11], Xu and Xia [68], Peng et al. [37], and Chen et al. [7] discussed the information measures of HFSs. Hesitant fuzzy TOPSIS [69] and hesitant fuzzy

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TODIM [80] methods for solving MCDM problems have been proposed. Chen et al. [8] extended HFSs to interval-valued hesitant fuzzy sets (IVHFSs), which represent the membership degrees of an element to a set with several possible interval values. Wei et al. [58] proposed that IVHFSs could be used to investigate MCDM problems. Farhadinia [11] investigated the relationship between the entropy, similarity measure, and distance measure for HFSs and IVHFSs. Wei et al. [59] and Peng et al. [38] developed some aggregation operators for IVHFSs and applied them to MCDM problems. It should be noted that an IVHFS permits the membership degrees of an element to a given set to have differing interval values, and when the upper and lower limits of these interval values are equal, IVHFSs become HFSs.

When faced with problems that are too complex or ill-defined to be solved by quantitative expressions, linguistic variables can be an effective tool because the use of linguistic information enhances the reliability and flexibility of classical decision models [30]. Linguistic variables have been studied in depth and used in many fields [19,33,35,42,50,60,73,78]. The linguistic variable could be a single linguistic term [77], or interval of linguistic terms, i.e. uncertain linguistic variables [63]. Rodríguez et al. [40,41] proposed hesitant fuzzy linguistic term sets (HFLTSs) that assess a linguistic variable by using several linguistic terms, and research in this area has been growing [3,20,22,25,42,51,55,82,83]. However, they can depict only one fuzzy attribute of an object and cannot reflect the possible membership degrees of a linguistic term to a given concept. Therefore, intuitionistic linguistic sets [54] and their extensions [26,27] have been proposed.

To express decision-makers' hesitance that exists in giving the associated membership degrees of one linguistic term, the concept of hesitant fuzzy linguistic sets (HFLSs), which are based on linguistic term sets and HFSs, was introduced in [23]. The elements in HFLSs are called hesitant fuzzy linguistic elements (HFLEs). For example, $<s_2, (0.3, 0.4, 0.5)>$ is an HFLE and 0.3, 0.4, and 0.5 are the possible membership degrees to the linguistic term s_2. HFLSs have already enabled great progress in describing linguistic information and to some extent may be considered to be an innovative construct. However, under most conditions, decision information is usually uncertain or fuzzy due to the increasing complexity of the environment and the vagueness of the inherent subjective nature of human thought; thus, crisp values are inadequate or insufficient to model real-life decision problems; it might not be flexible or convenient for decision-makers to exactly quantify their opinions with crisp numbers. A possible solution is to represent such membership degrees by interval values. Indeed, human evaluation information may be so represented, which permits the membership degrees having a set of possible interval values to represent the degrees that an evaluation object attaches to the linguistic term. So, a new class of fuzzy sets called interval-valued hesitant fuzzy linguistic sets (IVHFLSs) is established in this paper, which follows the methodology used by HFLSs. The new class is based on linguistic term sets and IVHFSs. IVHFLSs combine the advantages of both linguistic term sets and IVHFSs. The main advantage of IVHFLSs is that they can describe two fuzzy attributes of an object: a linguistic term and an interval-valued hesitant fuzzy element (IVHFE). The former provides an evaluation value, such as "excellent" or "good". The latter describes the hesitancy for the given evaluation value and denotes the interval-valued membership degrees associated with the specific linguistic term.

To date, several methods have been proposed for dealing with linguistic information, and the main ones will now be briefly discussed. (1) A method based on a transformation to fuzzy numbers by means of membership functions [6,9,16,18,21,62]. However, this method led to a certain degree of information loss in the transformation process, and it is difficult to choose the appropriate membership functions in practical decision-making applications. (2) A method based on symbols that made computations on the subscripts of linguistic terms and was easy to operate [5,10,44]. However, this approach performed the retranslation step as an approximation process to express the results in the initial term sets, which led to a lack of accuracy [15]. (3) A method based on the cloud model, which can correctly depict the uncertainty of a qualitative concept. This model has been successfully used [48,49,52]. (4) A method based on the 2-tuple linguistic representation model [14], which avoids the information distortion and loss that had hitherto occurred in linguistic information processing [17,39,56,61]. However, as was depicted by Martínez and Herrera [28], these approaches possess certain advantages, but this does not mean that they can successfully model and solve all types of problems.

Clearly in real world decision-making problems it is not realistic to use only one technique, because different criteria, attributes, and other factors are better suited to different types of modeling. In the method based on the 2-tuple linguistic representation model, there is a conversion and inverse conversion process. Motivated by this idea and taking into consideration the limitations in previous linguistic methods, we propose new linguistic scale functions to deal with linguistic translation issues under different semantic situations. These scale functions provide a higher degree of flexibility for modeling linguistic information.

In general, aggregation operators are important tools for dealing with information fusion in MCDM problems and are a research area of great interest throughout the world. In practical situations, decision-makers usually consider different criteria priorities. To deal with this issue, Yager [71] developed prioritized average (PA) operators by modeling the criteria priority on the weights associated with the criteria, which depend on the satisfaction of higher priority criteria. Yager [72] further focused on PA operators and proposed two methods for formulating this type of aggregation process. As is well known, the PA operator has many advantages over other operators. For example, the PA operator does not need to provide weight vectors and, when using this operator, it is only necessary to know the priority among the criteria. However, Yager [71] only discussed the criteria values and weights in the real number domain. Therefore our aim is to develop some prioritized aggregation operators for aggregating interval-valued hesitant fuzzy linguistic information.

The focus of this paper is on a type of MCDM problems where a criterion priority exists, referred to as a prioritized MCDM. Two PA operators for interval-valued hesitant fuzzy linguistic numbers (IVHFLNs) that are a special case of IVHFLSs will be proposed and extended to a grouping prioritized situation. The main use of these operators is to solve MCDM problems in