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Fuzzy reasoning method by optimizing the similarity of truth-tables

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ABSTRACT

This paper presents a new fuzzy reasoning method by optimizing the similarity of truth-tables (OS method for short). Its basic idea is to find a fuzzy set such that the truth-tables generated by the antecedent rule and the consequent rule are as similar as possible. Based on this idea, the principle of OS method and the fuzzy reasoning with OS method are given and discussed. And then the OS methods with certain similarity measure and several fuzzy implications are investigated. Finally, numerical examples are analyzed to compare the proposed method with compositional rule of inference (CRI) method.

1. Introduction

Fuzzy reasoning has been widely used in many fields, such as fuzzy control, fuzzy data mining, artificial intelligence, image processing and so on. The core and foundation of fuzzy reasoning is the ***fuzzy modus ponens*** (FMP) derived from the modus ponens (MP) in the classic logic.

Let a and b be logic formulas. Then the tautology $(a \wedge (a \to b)) \Rightarrow b$ is the most basic inference rule called modus ponens in the classical logic, which states that if both an implication and its hypothesis are known to be true, then the conclusion of this implication is true. FMP is the generation of the modus ponens within the framework of fuzzy logic to facilitate approximate reasoning. If given a conditional fuzzy proposition

$$p: \text{ If } x \text{ is } A, \text{ then } y \text{ is } B,$$

and a proposition

$$q: \; x \text{ is } A^*,$$

then FMP can be expressed by the following schema:

$$\begin{array}{lccc} \texttt{Rule} & A & \to & B \\ \texttt{Fact} & A^* & & \\ \hline \texttt{To determine} & & & B^* \end{array} \tag{1}$$

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where A and A^* are fuzzy sets defined on X, B and B^* are fuzzy sets defined on Y. It is similar to modus ponens, but the premise A^* is slightly different from A and thus the conclusion B^* is slightly different from B [11].

The first model of inference which handled fuzzy rules is the compositional rule of inference (CRI), proposed by Zadeh [33]. Its first step is to translate $A \to B$ into a fuzzy relation R, which is also called ***truth-table***, by the formula

$$R(x,y) = I[A(x), B(y)],$$

where I denotes a binary operation on $[0,1]$ representing a suitable fuzzy implication. And then obtain B^* by the equation

$$B^*(y) = \bigvee_{x\in X}[A^*(x) \wedge R(x,y)].$$

This equation can also be written in matrix form as

$$B^* = A^* \circ R,$$

where the operation $\circ$ is the $\vee - \wedge$ composition. Initially, the implication used by Zadeh is

$$I_Z(a,b) = (1-a) \vee (a \wedge b), \quad a,b \in [0,1].$$

At present, it has been extended to the general implication operator, and the $\wedge$ in the composition has been replaced by t-norm.

Fuzzy reasoning has been successfully applied to many fields by utilizing the CRI method. However, as pointed out by some authors [15,19,24–27], the CRI method has some imperfections. For example, the approach does not possess the reductivity property. That means if A^* is equal to A, we cannot always infer that B^* is equal to B as we expect. So the consequences inferred by CRI method do not always fit our intuition. Li [15] has proved that, mathematically, the ordinary fuzzy control method depending on CRI may be regarded as a certain interpolation method. Wang [27] pointed out that the composition operation in CRI method lacks clear logical sense.

To improve the CRI method, Wang [27] proposed the full implication triple I method of fuzzy reasoning. Its principle for FMP is to obtain the smallest fuzzy set of the universe Y such that the following formula attains the greatest value for any x in X and y in Y:

$$(A(x) \to B(y)) \to (A^*(x) \to B^*(y)).$$

Since the introduction of the triple I method, a number of authors have studied the interesting inference method. For example, Song and Wu [22] proposed reverse triple I method of fuzzy reasoning. Luo and Yao [17] studied the triple I algorithms based on Schweizer–Sklar operators. Zhao and Li [35] studied the reverse triple I method with Lukasiewicz implication operator. Tang and Yang constructed a symmetric implicational method which contains the full implication inference method as its particular case. Zheng et al. [36] extended the triple I method of fuzzy reasoning on intuitionistic fuzzy reasoning. As pointed out by He and Quan [10], however, triple I method has some drawbacks and it cannot be applied in fuzzy control.

Another important type of fuzzy inference method is based on similarity measures. In [24,25], Turksen and Zhong mentioned the shortcomings of CRI method, then proposed a similarity-based fuzzy reasoning method called approximate analogical reasoning schema (AARS). Later, in order to solve the problems in medical diagnosis, Chen [3,4] constructed two similarity-based fuzzy reasoning methods called matching function (MF) method and function T (FT) method. Yeung and Tsang [31,32] proposed three similarity-based fuzzy reasoning methods named as degree of subsethood (DS) method, inclusion and cardinality (IC) method, and equality and cardinality (EC) method. Wang et al. [28] extended the similarity measure to fuzzy similarity measure and proposed a fuzzy similarity inference scheme. The basic idea of similarity-based fuzzy reasoning methods is to obtain the inference result B^* by modifying the consequent B with a modification function based on the similarity between A and A^*. Compared with the CRI method, the similarity-based fuzzy reasoning methods do not require the construction of fuzzy relation. The results obtained by these methods, however, strongly depend on the similarity measure and the modification function.

In addition, there are some other fuzzy reasoning methods. For example, Takagi and Sugeno [23] proposed Takagi–Sugeno (TS) fuzzy model, whose rules have fuzzy antecedent parts and linear functions in the consequences. This method has been widely used in fuzzy control. Baldwin [1] presented a fuzzy truth value (FTV) based reasoning by using fuzzy truth value restrictions and an implication composition in truth space. Kóczy and Hirota [13,14] presented a fuzzy interpolative reasoning method to tackle the situations of sparse fuzzy rule bases. Gera [7] constructed a membership driven inference reasoning scheme to handle the situations that the rules and premises were expressed by combinations of sigmoid-like membership functions. Setnes et al. [21] proposed a fuzzy arithmetic-based interpolative reasoning method which combined the properties of both Mamdani and TS models. Zhang and Li [34] provided a variable weighted synthesis inference method by applying the principle of variable weighted synthesis in factor spaces theory to fuzzy inference. Castro et al. [2] presented a non-monotonic fuzzy reasoning method using averaging functions to handle the conflicts existed in fuzzy consequences. Watanabe et al. [29] proposed a mean-value-based functional reasoning in which the conclusion part consists of a function of mean-values on each membership function in the antecedent. Considering the temporal information, Maeda et al. [18] proposed a dynamical fuzzy reasoning method that has a time delay between premise and consequence.

At present, the most commonly used fuzzy reasoning methods in applications are still CRI method and TS fuzzy model. Although these two methods are very simple, they have significant disadvantages. In addition, according to our statistics, in