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Bipolar fuzzy graph representation of concept lattice

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ABSTRACT

Formal Concept Analysis (FCA) is a mathematical framework for knowledge processing tasks. FCA has been successfully incorporated into fuzzy setting and its extension (interval-valued fuzzy set) for handling vagueness and impreciseness in data. However, the analysis in such settings is restricted to unipolar space. Recently, some applications of bipolar information are shown in bipolar fuzzy graph, lattice theory as well as in FCA. The adequate analysis of bipolar information using FCA requires incorporation of bipolar fuzzy set and an appropriate lattice structure. For this purpose, we propose an algorithm for generating the bipolar fuzzy formal concepts, a method for (α, β)-cut of bipolar fuzzy formal context and its implications with illustrative examples.

1. Introduction

Formal Concept Analysis (FCA) was introduced by Wille [52] for knowledge discovery and representation tasks. FCA starts the analysis from a given formal context which comprises a set of formal objects, a set of formal attributes, and a binary relation between them. From a given context FCA investigates formal concepts, which is a pair of extent and intent representing a subset of objects with their common attributes, respectively. Concept lattice provides connection between investigated formal concepts as generalization and specialization, which plays a major role in knowledge processing tasks [44,45]. FCA became more popular in scientific community when its mathematical foundation was established by Ganter and Wille [26].

Burusco and Fuentes-Gonzalez [19] incorporated FCA into the fuzzy setting for handling vagueness and impreciseness in linguistics variables, and was further improved by Belohlavek [12–14]. Afterwards, fuzzy formal concepts and their lattice structure were applied by several researchers for knowledge processing tasks [15,44,45]. Very recently, Li and Tsai [37] discussed an application of the fuzzy concept lattice for sentiments (emotions, love, etc.) classification based on the opinion of people. Antoni et al. [6] introduced heterogeneous formal context for representing preference of people to stay at cottage in a given facility. Subsequently, Franco et al. [25] introduced a model for the preference analysis. The word opinion or preference shows two sides: one is the positive (acceptation) and the other is the negative (non-acceptation) side. These two sides coexist simultaneously, and can be represented as an integral of its positive and negative sides. In this case, we cannot apply the existing fuzzy approaches in FCA [15,20–22,24,27,30,31] because it defines the data in the unipolar space $\{0, 1\}$ or $[0, 1]$. An another extension of fuzzy set (called as bipolar fuzzy set) represents the bipolar information more precisely [58]. Considering the above scenario, it is important to introduce the bipolar fuzzy set into FCA for analyzing the bipolar information using concept lattice.

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Nomenclature

L	scale of truth degree
$\mathbf{L}$	residuated lattice
$(X, Y, \widetilde{R})$	fuzzy formal context – **F**
$\widetilde{R}$	A map from $X \times Y$ to L
$\otimes$	multiplication
$\rightarrow$	residuum
a, b, c	elements in L
$(\uparrow, \downarrow)$	galois connection
A	extent
B	intent
C	set of bipolar fuzzy formal concept.
L^X	L – set of objects
L^Y	L – set of attributes
$\bigcup$	union
$\bigcap$	intersection
$\wedge$	infimum
$\vee$	supremum
m	total number of attributes
n	total number of objects
$\|\|$	cardinality
$\mu^P(z)$	positive membership degree
$\mu^N(z)$	negative membership degree

A bipolar information consists two sides: one is positive, and another is negative side. For example, the relation between two organizations constitute a conflict side and a common interest side [60]. If we assume -1 to represent negative pole true, and 0 as false then 1 represents positive pole true, and 0 for false. This case can be represented adequately through a bipolar fuzzy set defined in a bipolar fuzzy space $[-1,0] \times [0,1]$ [58–60]. A bipolar fuzzy set is a pair of mappings, namely a positive membership $(0,1]$, and a negative membership function $[-1,0)$ [32]. The positive membership degree $(0,1]$ of an element indicates that the element somewhat satisfies the corresponding property, and the negative membership degree $[-1,0)$ of an element indicates that the element somewhat satisfies the implicit counter-property [1,2,55]. The zero membership degree $\{0\}$ of an element means the element is irrelevant to the corresponding property [34]. This representation is necessary because fuzziness is inseparable from bipolar truth [23,29]. Therefore, it is necessary to introduce it into fuzzy concept lattice.

Techniques available in the literature for visualizing the concept lattice are restricted to analyze the data in the unipolar space [4–16,19–22,24–27,29,33,35–37,39,42,46–50,56,57], which lacks in visualizing the bipolarity. Some applications of bipolar information were shown in lattice theory [6,17,18,20,21], information retrieval [38,51–56], as well as in bipolar fuzzy graph [1–3,28,40,53,54]. In this paper our analysis focus on visualizing the bipolar information using the concept lattice. To achieve this goal, we require an appropriate lattice structure, and graph theory for concept lattice representation. Recently, Bloch [17,18] discussed the lattices of bipolar fuzzy set, its properties and applications. Niesink et al. [40] defined the properties of weighted bipolar fuzzy graph. Akram [1–3] discussed several properties of bipolar fuzzy graphs with its applications followed by Yang et al. [54]. In this paper we focus on representation of concept lattice using the properties of bipolar fuzzy graph and lattices of bipolar fuzzy set. The motivation is to represent the positive and negative side of bipolar fuzzy attributes simultaneously in the concept lattice, and provide a more adequate analysis by the connected (bipolar) fuzzy formal concepts, in the form of generalization and specialization. Such that, the theory of concept lattice can commensurate with preference analysis [6,25], mathematical morphology [17,18], decision making [29], customer satisfaction [28,30], opinion classification [37] and handling bipolar queries [55]. In this process, we address the following problems:

(1) How to represent the bipolar information in fuzzy formal context?
(2) How to investigate the formal concepts hidden in given bipolar fuzzy context?
(3) How to visualize the hierarchical order between the generated bipolar fuzzy concepts into the concept lattice?
(4) How to find bipolar fuzzy attribute implications?

To address these problems, we aim at the following proposals in this paper:

(1) To introduce FCA with bipolar fuzzy setting.
(2) To introduce a method for investigating the bipolar fuzzy formal concepts.