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A risk index model for multi-period uncertain portfolio selection

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ABSTRACT

This paper discusses a multi-period portfolio selection problem when security returns are given by experts' evaluations. The security return rates are regarded as uncertain variables and an uncertain risk index adjustment model is proposed. Optimal portfolio adjustments are determined with the objective of maximizing the total incremental wealth within the constraints of controlling the cumulative risk index value over the investment horizon and satisfying self-financing at each period. To enable the users to solve the model problem with currently available programming tools, an equivalent of the model is provided. In addition, a method of obtaining the uncertainty distributions of the security returns is given based on experts' evaluations, and a selection example is presented.

1. Introduction

Portfolio selection involves optimal allocation of one's capital to a number of candidate securities. Since Markowitz [20], the idea of mean-variance has become a widely accepted research framework in portfolio selection not only over a single period [4,10,22] but also over a number of periods [2,7,25]. In the mean-variance framework, the expected return is regarded as the investment return. When the variance value is greater, the deviation level from the expected return is greater. This in turn implies that the investors are less likely to obtain the expected return. So variance is regarded as the investment risk. Though variance is a popular risk measurement, it is not very convenient to use for some investors. First, variance provides no information about how much money investors may lose, but it is the loss of their money that is the investors' concern. When the portfolio return rate is lower than 0 or a preset base profit, investors feel they are losing money. Usually it is easier for them to tell how much loss they can tolerate than how much variance they can tolerate. Second, in the traditional mean-variance framework, to ensure that the portfolio will be sufficiently safe, investors should first give their maximum tolerable variance level and ask that the variance value of the portfolios should not be greater than the preset tolerable variance level. Then they can select the portfolio with the maximum expected return. The investors need to give a maximum tolerable variance level before knowing the expected return of the portfolio. However, it is difficult to judge if a variance level is tolerable or not when the expected value of the portfolio is unknown, because with different expected values, the investors' maximum tolerable variance levels may be different. For example, investors may not tolerate variance value 1 for a portfolio with expected return 0, but may tolerate the same variance value 1 for a portfolio with expected value 10. This fact makes variance even harder to use as a risk measurement. To overcome the difficulty of pre-giving a reasonable variance value, Sharpe ratio was developed. This is calculated by subtracting the risk-free rate from the expected portfolio return rate and dividing the result by the standard deviation of the portfolio return rates. However, Sharpe ratio still does not give investors recognizable information about how much money they may lose. To provide an easy to use risk measurement and to give investors direct

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and easily recognizable information about how much money they may lose, we will propose a new risk measurement from a new perspective. Since in real life there are cases when historical data can hardly reflect the future performances of security returns and since the predictions of security returns are made mainly based on experts' evaluations, we will employ uncertainty theory to help solve a multi-period portfolio selection problem based on the new risk measurement in such a situation.

The rest of the paper is organized as follows. Section 2 will first justify the use of uncertain variables in portfolio selection with returns subject to experts' judgements and then briefly review some properties of uncertain variables which will be used in the paper. Section 3 will discuss a multi-period investment process and propose a mean-risk index model. In order to solve the uncertain model, Section 4 will give an equivalent of the model when security returns are all normal uncertain variables, and Section 5 will provide a Delphi method to obtain the expected and standard deviation values of the uncertain security returns. To illustrate the application of the proposed model, Section 6 will present a selection example. Finally, Section 7 will give some concluding remarks.

2. Uncertain variables in imprecise estimations

In reality, it is found that sometimes the prediction of security returns has to rely on experts' evaluations rather than historical data because of the complexity of the security market. Therefore, many scholars have argued that in this situation randomness is not how security returns behave. With the introduction of fuzzy set theory by Zadeh [26] in 1965, fuzzy numbers were employed to describe subjective imprecise quantity. An early attempt at using fuzzy set theory to handle portfolio selection problems was done by Watada [24] in 1997. He used fuzzy numbers to represent the decision makers' aspiration levels for the expected rate of return and a certain degree of risk. Later on, many scholars studied how to use the mean-variance framework to select the portfolio in this situation by using fuzzy set theory, and different versions of fuzzy mean-variance models have been developed, e.g. possibilistic models by Carlsson et al. [3], Gupta et al. [10], Zhang et al. [27], and credibilistic models by Huang [11,12], Qin et al. [21], and Li et al. [17], etc. Recently, Bilbao-Terol et al. [1] has used goal programming and fuzzy technology to select a socially responsible portfolios.

This research opened the way to handling portfolio selection problems with subjective imprecise return rate estimations. However, further research has found that paradoxes will appear if we use fuzzy variables to describe the subjective imprecise estimations of security returns. For example, if we regard a security return rate as a fuzzy variable, then we should have a membership function to characterize it. Suppose it is a triangular fuzzy variable $\xi = (-0.2, 0.4, 1.0)$. Based on the membership function, we know from possibility theory (or credibility theory) that $\text{Pos}\{\xi = 0.4\} = 1$ (or $\text{Cr}\{\xi = 0.4\} = 0.5$), which means that *the return rate is exactly 0.4 with a belief degree 1 in possibility measure (or 0.5 in credibility measure)*. However, this conclusion is unreasonable because the belief degree of *exactly 0.4* should be almost zero. Furthermore, we get from possibility theory that $\text{Pos}\{\xi = 0.4\} = \text{Pos}\{\xi \neq 0.4\}$ (or from credibility theory that $\text{Cr}\{\xi = 0.4\} = \text{Cr}\{\xi \neq 0.4\}$), which means that the return rate being *exactly 0.4* and *not exactly 0.4* have the same belief degree in either possibility measure or credibility measure. In other words, the result implies that these two events of the return rate being *exactly 0.4* and *not exactly 0.4* will be equally likely to happen. This conclusion is also hard to accept. In order to solve these problems and to model the subjective imprecise quantity, Liu proposed an uncertain measure and uncertain variable [18] and further developed an uncertainty theory [19] based on an axiomatic system of normality, self-duality, countable subadditivity, and product uncertain measure. When we use the uncertain variable to model the experts' imprecise estimations of security returns, the above mentioned paradoxes disappear. Nowadays, uncertainty theory has been employed in many theoretical studies and applications. For example, Chen et al. [5] studied the properties of cross entropy based on uncertainty theory. Zhu [29] developed an uncertain optimal control theory and applied it to portfolio selection. Huang proposed a mean-risk curve [14], a mean-variance method [15], and a risk index method [16] to deal with portfolio optimization with returns given by experts' judgements. Other portfolio selection methods based on uncertainty theory can be found in [13]. In addition, Gao [9] solved a shortest path problem with arc lengths offered by experts' evaluations, and Zhang et al. [28] proposed a multi-national project selection method with project parameters given by experts' judgements. Other applications of uncertainty theory can be found in the areas of subjective uncertain logic [6], subjective uncertain inference [8], etc. In this paper, we will use uncertain variable to describe the experts' estimations of security return rates. As discussed in the Introduction, variance is not a convenient risk measurement for some investors in a random environment because it gives no information about how much money they may lose. This is also true in a subjective uncertain environment. Therefore, we here introduce a risk index as an alternative risk measurement and employ uncertainty theory to solve a multi-period portfolio selection problem. For better understanding of the paper, we first briefly review some concepts and properties of uncertain variables which will be used in the paper.

Definition 1. Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is called an event. A set function $\mathcal{M}\{\Lambda\}$ is called an uncertain measure if it satisfies the following three axioms (Liu [18]):

(i) (Normality) $\mathcal{M}\{\Gamma\} = 1$.
(ii) (Self-duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$.
(iii) (Countable subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have