Information Sciences 217 (2012) 117–126

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Sliding mode control for stochastic systems subject to packet losses

Tinggang Jia [a,b], Yugang Niu [a,*], Yuanyuan Zou [a]

[a] Key Laboratory of Advanced Control and Optimization for Chemical Processes, East China University of Science and Technology, Ministry of Education, 130 Meilong Road, Shanghai 200237, PR China
[b] Shanghai Electric Group Co. Ltd., Shanghai, PR China

ARTICLE INFO

Article history:
Received 11 April 2011
Received in revised form 25 April 2012
Accepted 10 June 2012
Available online 27 June 2012

Keywords:
Stochastic system
Packet dropout
Sliding mode control
Reachability

ABSTRACT

This paper investigates the problem of sliding mode control for stochastic system under the effect of packet dropout. In this work, a new sliding function dependent on the probability of packet dropout is constructed. By means of a Lyapunov function involved in sliding mode variable and system state, both the reachabiltiy and the stability are analyzed simultaneously, and the sufficient condition is derived via linear matrix inequalities. Finally, numerical simulation results are given.

1. Introduction

In the past decades, stochastic modeling has played an important role in many real-world systems, including nuclear, thermal, chemical processes, biology, social-economics, and so on. A lot of results related to robust stability and stabilization for stochastic systems have been reported, see [20,21,25,26,16] and the references therein. More recently, sliding mode control (SMC) for stochastic systems has received increasing attention in [8,1,12,6,17]. The notable advantage of SMC systems is its strong robustness against uncertainties and external disturbance, which makes it become an effective control strategy for incompletely modeled (or uncertain) systems [24,23]. However, it is noted that in the aforementioned works, the problem of data packet dropout has not been considered. That is, the system signals could be successfully transmitted to the controller or actuator. Actually, the above case may be only true for those traditional point-to-point control systems.

As well-known, with the rapid development of computer network technology, more and more system information is transmitted via communication networks. Compared with point-to-point cases, the insertion of communication networks brings great advantages, such as low cost, reduced weight and power, simple installation and maintenance, and high reliability. In the meantime, some detrimental phenomena, e.g., induced time delay, data packet losses, etc., also inevitably rise, which may deteriorate the performance or even de-stabilize the systems [11,5,13,2,10]. Recently, a lot of efforts have been made on the design of control systems in the presence of data packet dropout in [19,3,7] and the reference therein.

However, to the authors' best knowledge, the design problem on sliding mode control for *stochastic* systems subject to packet losses still remains open. In particular, due to the characteristic of the structure of SMC and *stochastic* systems, these existing methods on SMC without packet dropout cannot be trivially extended to the case subject to packet dropout, which may also be seen from the design of sliding function later. This motivates the present study.

* Corresponding author.
E-mail addresses: tgjia@163.com (T. Jia), acniuyg@ecust.edu.cn (Y. Niu).

0020-0255/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.ins.2012.06.020

In this work, the problem of SMC for *stochastic* systems is considered, in which the packet losses may happen during the transmission of the state signals from the sensor to the controller. To simplify, it is assumed that the packet dropout may not happen successively, i.e., only single packet dropout may happen. In order to cope with the dropout of data packet, a compensating method is introduced by replacing the lost packet with the last received one. The basic idea of SMC is to drive the state trajectory of the system onto some specified sliding surface (passing through the zero state) or into its neighborhood and maintains the trajectory within it for all subsequent time. To this end, an integral-like sliding surface*involving* dropout probability is constructed in this work. It is known that in the discrete-time case, the control input is computed at each sampling instant and is held as constant during the sampling interval. Thus, the system state in discrete-time SMC (DSMC) may approach the sliding surface but is generally unable to stay on it. As a result, it will move about the surface, which yields a sliding-like mode, i.e., *quasi-sliding* mode (QSM) [4]. Due to the characteristic of stochastic systems subject to packet dropout, it is difficult to directly analyze the reachability of QSM (as stated in Remark 2 later). Hence, in this work, the analysis on both the reachabiltiy and the stability of system states will be made *simultaneously*, which also makes the proposed method in this work different from some existing works. It is shown that, by means of the present SMC law, the state trajectories may be driven (in mean-square) into the band of the specified sliding surface despite packet losses. Moreover, the stability of ideal sliding motion is also ensured.

Notations: Throughout the paper, $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space with Ω the sample space, $\mathcal{F}$ the σ-algebra of subsets of the sample space, and $\mathcal{P}$ the probability measure. $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to probability measure $\mathcal{P}$. For a real symmetric matrix, $M > 0$ (< 0) means that M is positive-definite (negative-definite). I is used to represent an identity matrix of appropriate dimensions. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of a matrix, respectively. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following discrete-time stochastic system:

$$x(k+1) = Ax(k) + B(u(k) + f(x(k), k)) + (C + \Delta C(k))x(k)w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^m,\ w(k)$ is a scalar Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_k)_{k\in\mathbb{N}}$ of σ-algebra $\mathcal{F}_k \subset \mathcal{F}$ generated by $(w(k))_{k\in\mathbb{N}}$ with $\mathbb{N}$ the set of natural numbers, and is assumed to satisfy $\mathcal{E}\{w(k)\} = 0,\ \mathcal{E}\{w(k)^2\} = 1$, and $\mathcal{E}\{w(i)w(j)\} = 0$ for $i \neq j$. A, B, and C are known real constant matrices. Without loss of generality, it is assumed that the pair (A,B) is controllable and the input matrix B has full column rank. The unknown matrix $\Delta C(k)$ represents uncertainty satisfying $\Delta C(k) = EF(k)H$, where E and H are known real constant matrices, and $F(k)$ is an unknown time-varying matrix satisfying $F^T(k)F(k) \leqslant I,\ \forall t$. The unknown nonlinear function $f(x(k), k)$ is the external disturbance with known constant bound.

Definition 1. For the stochastic system (1), if there exist constants $\alpha > 0$ and $\tau \in (0, 1)$ such that

$$\mathcal{E}\{||\eta(k)||^2\} \leqslant \alpha\tau^k\mathcal{E}\{||\eta(0)||^2\}$$

where $\eta(k)$ denotes the solution of stochastic systems with initial state $\eta(0)$, the stochastic system (1) is said to be exponentially mean-square stable.

In this work, the system state signals are transmitted to the controller via communication network, in which the dropout of data packet will inevitably happen. It is assumed that the probability distribution of the packet dropout obeys Bernoulli process $\theta \in R$ as follows:

$$\mathcal{P}\{\theta = 1\} = \bar{\theta}, \quad \mathcal{P}\{\theta = 0\} = 1 - \bar{\theta}, \tag{2}$$

where $0 \leqslant \bar{\theta} < 1$ represents the probability that any data packet will be lost. Moreover, to simplify, this work only considers the case that the dropout of only single packet may happen. Thus, in order to compensate the lost packet, the following method will be utilized in this work:

$$x_p(k) = (1 - \theta)x(k) + \theta x(k-1). \tag{3}$$

Now, the objective of this work is to design an SMC law for stochastic system (1) such that the resultant closed-loop system is exponentially mean-square stable despite packet losses and external disturbance. To this end, a sliding surface will be firstly chosen, and the SMC law will be designed such that the reachability of specified sliding surface can be ensured. Besides, the stability of ideal sliding mode in the specified sliding surface will be also analyzed.

3. Sliding mode control and reachability

As discussed in Section 2, the packet dropout (2) may happen in the stochastic system (1). Thus, by considering the probability of packet losses, an integral-like sliding function is constructed as follows:

$$s(k) = (1 - \bar{\theta})Gx(k) + \bar{\theta}GAx(k-2), \tag{4}$$