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The 2-path-bipanconnectivity of hypercubes ☆

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ABSTRACT

In this paper, we introduce the concept of the k-path-(bi)panconnectivity of (bipartite) graphs. It is a generalization of the (bi)panconnectivity and of the paired many-to-many k-disjoint path cover. The 2-path-bipanconnectivity with only one exception of the n-cube Q_n ($n \geqslant 4$) is proved. Precisely, the following result is obtained: In an n-cube with $n \geqslant 4$ given any four vertices u_1, v_1, u_2, v_2 such that two of them are in one partite set and the another two are in the another partite set. Let $s = t = 5$ if $C = u_1u_2v_1v_2$ is a cycle of length 4, and $s = d(u_1, v_1) + 1$ and $t = d(u_2, v_2) + 1$ otherwise, where $d(u, v)$ denotes the distance between two vertices u and v. And let i and j be any two integers such that both $i - s \geqslant 0$ and $j - t \geqslant 0$ are even with $i + j \leqslant 2^n$. Then there exist two vertex-disjoint (u_1, v_1)-path P and (u_2, v_2)-path R with $|V(P)| = i$ and $|V(R)| = j$. As consequences, many properties of hypercubes, such as bipanconnectivity, bipanpositionable bipanconnectivity [18], bipancycle-connectivity [12], two internally disjoint paths with two given lengths, and the 2-disjoint path cover with a path of a given length [21], follow from our result.

1. Introduction

One of the central issues in various interconnection networks is to find node-disjoint paths concerned with the routing among nodes. The problem of node-disjoint paths has received much attention because of its numerous applications in high-performance communication, fault-tolerant routing, and so on. In this paper, when we mention disjoint paths to mean node-disjoint paths. We follow [1] for terminology and notation on graph theory.

Given any two disjoint sets of k labeled vertices $S = \{u_1, u_2, \ldots, u_k\}$ and $T = \{v_1, v_2, \ldots, v_k\}$ in a graph G, called sources and sinks, respectively. If there exist k disjoint paths $P_1, P_2, \ldots, P_k$ in G, where P_i connects u_i and v_i for $i = 1, 2, \ldots, k$, and they contain all vertices of G, then G is said to have paired many-to-many k-disjoint path cover [25]. If a graph G has paired the many-to-many 1-disjoint path cover, then there exists a Hamiltonian path between its any two vertices, and the graph G is called Hamiltonian-connected. Clearly, the paired many-to-many k-disjoint path cover is a generation of Hamiltonian-connectivity. If there exist k disjoint paths $P_1, P_2, \ldots, P_k$ containing all vertices of G, such that one end-vertex of P_i is in S and the another end-vertex is in T for each $i = 1, 2, \ldots, k$, then G is said to have unpaired many-to-many k-disjoint path cover [25]. For $k \geqslant 2$, the problem of the (un)paired many-to-many k-disjoint path cover for several classes of networks has been investigated, see [2,5,6,10,14,20,22,24–26].

Let u and v be any two vertices of a (bipartite) graph G. If there exists a path between u and v of every length l such that $d(u, v) \leqslant l \leqslant |V(G)| - 1$ (and $l - d(u, v)$ is even), where $d(u, v)$ is the distance between u and v, then the graph G is called (bi)panconnected. The (bi)panconnectivity of many classes of networks has been investigated.

We now give the following definitions.

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Definition 1. Given any two disjoint sets of k labeled vertices $S = \{u_1, u_2, \ldots, u_k\}$ and $T = \{v_1, v_2, \ldots, v_k\}$ in a graph G. Let $s_i = d(u_i, v_i) + 1$ and let r_i be any integer, such that $r_i \geqslant s_i$ for $i = 1, 2, \ldots, k$, and $\sum_{i=1}^{k} r_i \leqslant |V(G)|$. If there exist k disjoint paths $P_1, P_2, \ldots, P_k$ in G where P_i connects u_i and v_i and $|V(P_i)| = r_i$ for $i = 1, 2, \ldots, k$, then we say that G is k-path-panconnected.

Definition 2. Let G be a bipartite graph with partite sets B and W of the same size. Given any two disjoint sets of k labeled vertices $S = \{u_1, u_2, \ldots, u_k\}$ and $T = \{v_1, v_2, \ldots, v_k\}$ in a graph G such that $|(S \cup T) \cap B| = |(S \cup T) \cap W| = k$. Let $s_i = d(u_i, v_i) + 1$, and let r_i be any integer, such that $r_i - s_i \geqslant 0$ is even for $i = 1, 2, \ldots, k$, and $\sum_{i=1}^{k} r_i \leqslant |V(G)|$. If there exist k disjoint paths $P_1, P_2, \ldots, P_k$ in G, where P_i connects u_i and v_i and $|V(P_i)| = r_i$ for $i = 1, 2, \ldots, k$, then we say that G is k-path-bipanconnected.

Clearly, the k-path-(bi)panconnectivity of a (bipartite) graph is a generalization of the (bi)panconnectivity and is also a generalization of the paired many-to-many k-disjoint path cover.

It is well known that the n-dimensional hypercube (or the n-cube), denoted by Q_n, is one of the most popular and efficient interconnection networks. It possesses many excellent properties such as recursive structure, vertex and edge symmetry, maximum connectivity, bipanconnectivity, and easy routing algorithms. There is a large amount of literature on topological properties of hypercubes (with faulty elements), see [2–18,21–23,27–32,34–39].

Let u_1, v_1, u_2, v_2 be four vertices in the n-cube Q_n such that $C = u_1u_2v_1v_2$ is a cycle of length 4. Clearly, a path between u_1 and v_1 of length 2 must contain u_2 or v_2. In this paper, we are to show that this is the only one exception of the 2-path-bipanconnectivity of the n-cube Q_n, precisely, we are to show the following two theorems.

Theorem 1. *In an n-cube $Q_n (n \geqslant 4)$ given any four vertices u_1, v_1, u_2, v_2 such that u_i and v_i are in different partite sets for $i = 1, 2$. Let $s = d(u_1, v_1) + 1$ and $t = d(u_2, v_2) + 1$, and let i and j be any two even integers such that $s \leqslant i \leqslant 2^n - t$, $t \leqslant j \leqslant 2^n - s$ and $i + j \leqslant 2^n$. Then there exist two disjoint (u_1, v_1)-path P and (u_2, v_2)-path R with $|V(P)| = i$ and $|V(R)| = j$.*

Theorem 2. *In an n-cube $Q_n (n \geqslant 4)$ given any four vertices u_1, v_1, u_2, v_2 such that u_1 and v_1 are in one partite set, and u_2 and v_2 are in the another partite set. Let $s = t = 5$ if $C = u_1u_2v_1v_2$ is a cycle of length 4, and $s = d(u_1, v_1) + 1$ and $t = d(u_2, v_2) + 1$ otherwise. And let i and j be any two odd integers such that $s \leqslant i \leqslant 2^n - t$, $t \leqslant j \leqslant 2^n - s$ and $i + j \leqslant 2^n$. Then there exist two disjoint (u_1, v_1)-path P and (u_2, v_2)-path R with $|V(P)| = i$ and $|V(R)| = j$.*

As consequences, many properties of hypercubes, such as bipanconnectivity, bipanpositionable bipanconnectivity [18], bipancycle-connectivity [12], two internally disjoint paths with two given lengths, and the 2-disjoint path cover with a path of a given length [21] follow from our result.

The rest of this paper is organized as follows. In the next section, we present some preliminaries and lemmas. In Sections 3 and 4, the proof of Theorem 1 and of Theorem 2 is given, respectively. In Section 5, some corollaries are derived. Finally, the concluding remarks of this paper are presented in Section 6.

2. Preliminaries and lemmas

Throughout this paper, a graph $G = (V, E)$ means a simple graph, where $V = V(G)$ is its vertex-set and $E = E(G)$ is its edge-set. We use $P = (v_0, v_1, \ldots, v_k)$ to denote a (v_0, v_k)-path with k edges, where the two vertices v_0 and v_k are called its end-vertices. If $P = (v_0, \ldots, v_i, \ldots, v_j, \ldots, v_k)$ is a path, then $P[v_i, v_j] = (v_i, \ldots, v_j)$ is called a subpath of the path P. We use $C = v_1v_2 \cdots v_k$ to denote a cycle with $k(\geqslant 3)$ edges. The number of the edges in a path P (respectively, cycle C) is called its length and denoted by $l(P)$ (respectively, $l(C)$). The length of a shortest path between two vertices u and v is called their distance, and denoted by $d(u, v)$. A path (respectively, cycle) containing all vertices of a graph is called its Hamiltonian path (respectively, Hamiltonian cycle). Let G be a graph and $E' \subset E(G)$, $V' \subset V(G)$. We use $G - E'$ to denote the graph obtained from G by deleting all edges of E', and $G - V'$ to denote the graph obtained from G by deleting all vertices of V' and all edges incident with V'. Let P_1 and P_2 be two paths intersecting in only a common end-vertex, we use $P_1 \cup P_2$ to denote the path with $V(P_1 \cup P_2) = V(P_1) \cup V(P_2)$ and $E(P_1 \cup P_2) = E(P_1) \cup E(P_2)$.

The n-cube is a graph with 2^n vertices, and its any vertex v is denoted by an n-bit binary string $v = \delta_n\delta_{n-1} \cdots \delta_2\delta_1$, where $\delta_i \in \{0, 1\}$ for all i, $1 \leqslant i \leqslant n$. Two vertices of Q_n are adjacent if and only if their binary strings differ in exact one bit position. The Hamming distance $H(u, v)$ between two vertices u and v in Q_n is the number of different bits in the corresponding strings of the two vertices. Let $B = \{v \in V(Q_n) | H(v, O) \text{ is even}\}$ and $W = \{v \in V(Q_n) | H(v, O) \text{ is odd}\}$, where O is the vertex whose every bit is 0. It is clear that the Hamming distance $H(u, v)$ is just the distance $d(u, v)$ between two vertices u and v in Q_n, and Q_n is an n-regular bipartite graph with partite sets B and W. Assume $e = (u, v)$ is an edge of Q_n and the two binary strings of u and v differ in the ith bit position, then e is called an edge of dimension i in Q_n. The set of all edges of dimension i in Q_n is denoted by E_i. For any given $j \in \{1, 2, \ldots, n\}$, let Q_{n-1}^0 and Q_{n-1}^1 be two $(n-1)$-dimensional subcubes of Q_n induced by all vertices with the jth bit being 0 and 1, respectively. Clearly, Q_n is partitioned into Q_{n-1}^0 and Q_{n-1}^1 by E_j, we denote it by $Q_n - E_j = Q_{n-1}^0 \cup Q_{n-1}^1$. For a given $\delta \in \{0, 1\}$, if v is a vertex of Q_{n-1}^{δ}, then there is exactly one corresponding vertex in $Q_{n-1}^{1-\delta}$, denoted by $\bar{v}$, such that $(v, \bar{v}) \in E_j$, and if (u, v) is an edge in Q_{n-1}^{δ}, then there is exactly one corresponding edge $(\bar{u}, \bar{v})$ in $Q_{n-1}^{1-\delta}$ such that $(u, \bar{u}), (v, \bar{v}) \in E_j$.

For convenience, we give the following three definitions.