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A fuzzy multi-criteria decision-making model by associating technique for order preference by similarity to ideal solution with relative preference relation

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ABSTRACT

Generally, classical multi-criteria decision-making (MCDM) methods were extended to encompass uncertainty and vagueness of messages under fuzzy environment for solving decision-making problems, especially for technique for order preference by similarity to ideal solution (TOPSIS). In the fuzzy extension of TOPSIS, fuzzy numbers comparison and aggregation based on fuzzy preference relation are important issues to compute distance values between alternatives and ideal (or anti-ideal) solution or rank feasible alternatives, because lots of messages are reserved by fuzzy preference relation. However, fuzzy preference relation on pair-wise comparison is commonly too complex to calculate. To avoid the drawback, we use a relative preference relation improved from fuzzy preference relation in the fuzzy extension of TOPSIS for computing distance values between alternatives and ideal (or anti-ideal) solution, or obtaining relative closeness coefficients of alternatives. Thus the relative preference relation on fuzzy numbers will be associated with TOPSIS under fuzzy environment to develop a fuzzy multi-criteria decision-making (FMCDM) model. Through the association above, FMCDM problems can be easily solved by the model. Further, we compare the proposed model with other methods to demonstrate the model's feasibility and rationality.

1. Introduction

Decision-making is one of important issues for enterprises because the issue is to find an optimal alternative from a number of feasible alternatives. Further, decision-making with several evaluation criteria is named multi-criteria decision-making (MCDM) [1–5,7–9,11,13,14,16–19,21–23,25,27–42,44–46,48–62]. A MCDM model is commonly expressed in matrix format as follows.

$$G = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \dots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{array}$$

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and

$$W = [W_1, W_2, \ldots, W_n],$$

where $A_1, A_2, \ldots, A_m$ are feasible alternatives, $C_1, C_2, \ldots, C_n$ are evaluation criteria, G_{ij} is the evaluation rating of A_i on C_j, and W_j is the weight of C_j.

MCDM problems are practically classified into two categories. One is classical MCDM problems such as [25,29,34]. The other is fuzzy multi-criteria decision-making (FMCDM) problems such as [3,5,8,9]. In the classical MCDM problems, evaluation ratings and criteria weights on certain environment are expressed by crisp values. In the FMCDM problems, evaluation ratings and criteria weights are assessed on imprecision, subjectivity or vagueness, so the ratings and weights are often presented by linguistic terms [20,26] and then transformed into fuzzy numbers [67,69,70]. For instance, the evaluation ratings may be described by very poor (VP), poor (P), medium poor (MP), fair (F), medium good (MG), good (G) and very good (VG), and the criteria weights are denoted by very low (VL), low (L), medium (M), high (H) and very high (VH). Since the linguistic terms above can be transformed into fuzzy numbers, the evaluation is viewed as a FMCDM problem.

Many past researchers applied classical MCDM methods under fuzzy environment [3–5,7–9,11,13,16,17,22,23,27,28,30,31,33,35–42,44–46,48–50,52–62] to solve FMCDM problems. The approaches are commonly classified into two categories which are defuzzification and fuzzy extension. Defuzzification often lost lots of messages, so Chen [9], Liang [42], Raj and Kumar [49], Wang et al. [62] supposed that classical MCDM methods, such as technique for order preference by similarity to ideal solution (TOPSIS) [29], should be extended under fuzzy environment, i.e. fuzzy extension of TOPSIS [7–9,11,22,30,35–38,45,46,50,53,55–58,60,62]. TOPSIS proposed by Hwang and Yoon is one of well-known classical MCDM methods and often generalized under fuzzy environment into fuzzy extension of TOPSIS. The underlying logic of TOPSIS is to define ideal solution and anti-ideal solution. The ideal solution is a solution that maximizes benefit criteria and minimizes cost criteria, whereas the anti-ideal solution is a solution that maximizes cost criteria and minimizes benefit criteria. In short, the ideal solution is composed of all best values on criteria, and the anti-ideal solution consists of all worst values on criteria. Thus the optimal alternative has the shortest distance to the ideal solution and the farthest distance to the anti-ideal solution for all feasible alternatives.

In numerous extensions of TOPSIS, approaches of Chen [9], Liang [42], Raj and Kumar [49], Wang et al. [62] are useful for FMCDM. However, there were some drawbacks in their works. For instance, Liang [42], Raj and Kumar [49] utilized maximizing and minimizing sets [10] to rank evaluated values which were trapezoidal fuzzy numbers multiplied and then added. Since two trapezoidal fuzzy numbers multiplied would not be a trapezoidal fuzzy number, the raking for the type of fuzzy numbers was complex and difficult. Moreover, distance values from two varied alternatives to ideal solution (or anti-ideal solution) might be indiscernible, when intersections of the two alternatives and the ideal (or anti-ideal) solution on all criteria were null. Chen [9] used $(0,0,0)$ and $(1,1,1)$ to be respectively the worst and best values of alternatives on criteria. The two values might far from away minimum and maximum values in FMCDM problems, so $(0,0,0)$ and $(1,1,1)$ did not stand for the worst and best values of alternatives on criteria. Besides, multiplying ratings by weights (i.e. weighted ratings) were presented by triangular fuzzy numbers in Chen's method as ratings and weights were triangular fuzzy numbers. However, a weighted rating being product of two triangular fuzzy numbers was not a triangular fuzzy number. Practically, these problems can be solved by defuzzification or fuzzy preference relation. Through previous description, messages are often lost on defuzzification. Additionally, fuzzy operation by preference relation [4,6,33,34,36,41,43,44,47,54,63–66,68] is a hard work because fuzzy numbers are pair-wise compared under fuzzy environment. Thus we propose a FMCDM model based on TOPSIS and relative preference relation to resolve these ties. The relative preference relation is an improvement of fuzzy preference relation, and does not pair-wise compare fuzzy numbers under fuzzy environment.

For the sake of clarity, mathematical preliminaries are expressed in Section 2. The relative preference relation on fuzzy numbers is presented in Section 3. In Section 4, a FMCDM model based on TOPSIS and relation preference relation is proposed. A numerical example constructed on the FMCDM model is illustrated in Section 5. Finally, feasibility and rationality of the proposed model is demonstrated in Section 6.

2. Mathematical preliminaries

In this section, we view notions of fuzzy sets and fuzzy numbers [67,69,70] expressed as follows.

Definition 2.1. Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 2.2. α-*cut* of the fuzzy set A is a crisp set $A_\alpha = \{x | \mu_A(x) \geqslant \alpha\}$.

Definition 2.3. Support of the fuzzy set A is a crisp set $Supp(A) = \{x | \mu_A(x) > 0\}$.

Definition 2.4. The fuzzy set A of U is normal iff $\sup_{x \in U} \mu_A(x) = 1$.