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Objective space partitioning using conflict information for solving many-objective problems

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ABSTRACT

We present an algorithm that partitions the objective space based on an analysis of the conflict information obtained from the current Pareto front approximation. By partitioning the objectives in terms of the conflict among them, we aim to separate the multiobjective optimization into several subproblems in such a way that each of them contains the information to preserve as much as possible the structure of the original problem. We implement this framework by performing ranking and parent selection independently in each subspace. Our experimental results show that the proposed conflict-based partition strategy outperforms a similar algorithm in a test problem with independent groups of objectives. In addition, the new strategy achieves a better convergence and distribution than that produced by a strategy that creates subspaces at random. In problems in which the degree of conflict among the objectives is significantly different, the conflict-based strategy presents a better performance.

1. Introduction

Since the first implementation of a Multiobjective Evolutionary Algorithm (MOEA) in the mid 1980s [37], a wide variety of new MOEAs have been proposed, gradually improving in both their effectiveness and efficiency to solve Multiobjetive Optimization Problems (MOPs) [9]. However, until recently, most of these algorithms had been evaluated and applied to problems with only two or three objectives, in spite of the fact that many real-world problems have more than three objectives (e.g., see [18,25,39]).

Recent experimental [23,42,32,34] and analytical [40,27] studies have shown that MOEAs based on Pareto optimality [31] scale poorly when the number of objectives is increased. Although some scalability issues are known to mainly affect Pareto-based MOEAs (see e.g., [27,16,26]), optimization problems with a high number of objectives (also known as *many-objective problems*) introduce some difficulties common to any other multiobjective optimizer (e.g., visualization of the Pareto front). Three of the most serious difficulties are the following: (i) Deterioration of the search ability because the proportion of non-dominated solutions in a population increases rapidly with the number of objectives [16]. As a consequence, in many-objective problems, the selection of solutions is carried out almost at random or guided by diversity criteria only. (ii) The number of points required to achieve a representative sample of a Pareto front increases exponentially with the number of objectives.

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(iii) The visualization of the Pareto front is more complicated since with more than three objectives it is not possible to plot the Pareto front as usual. This is a serious problem since visualization plays a key role for a proper decision making.

Currently, there are three main approaches to solve many-objective problems, namely:

1. Adopt or propose an optimality relation that yields a solution ordering finer than that yielded by Pareto optimality. Among these alternative relations we can find k-optimality [16], preference order ranking [14], and a method that controls the dominance area [35]. Besides providing a richer ordering of the solutions, these relations obtain an optimal set which is a subset of the Pareto optimal set. Therefore, these techniques can be used as a partial remedy for the first and second issues of the previous enumeration.
2. Reduce the number of objectives of the problem during the search process [6,30] or, a posteriori, in the decision making process [36,4,29]. The main goal of this kind of reduction techniques is to identify the redundant objectives (or redundant to some degree) in order to discard them. A redundant objective is one that can be removed without changing the dominance relation[1] induced by the original objective set.
3. Incorporation of preference information interactively throughout the course of the optimization process [11,41,17]. By incorporating preferences, the search can be focused on the decision maker's region of interest, avoiding this way, the evaluation of a huge number of solutions.

A general scheme for partitioning the objective space into several subspaces in order to deal with many-objective problems was introduced in [3,2]. In that study we investigated the following three strategies to partition the objective space in equally sized subspaces: random (objectives for each subspace are assigned at random), fixed (objectives for each subspace are assigned sequentially), and shift (at each generation, objectives in the partition are shifted one position to the right). Here, we propose a new partition strategy that creates objective subspaces based on the analysis of the conflict information obtained from the Pareto front approximation found by the underlying MOEA. Additionally, we introduce a new version of the general partitioning scheme in order to improve the distribution along the Pareto front. By grouping objectives in terms of the conflict among them, we aim to separate the MOP into several subproblems in such a way that the union of these independent subproblems contains the information to preserve as much as possible the structure of the original problem.

In order to evaluate the effectiveness of the new conflict-based partition strategy, we compare its performance against different strategies. First, against a similar partition strategy proposed by Purshouse and Fleming [33] which is based on the Kendall correlation. We also compared the new partitioning approach with the random strategy and the original Non-dominated Sorting Genetic Algorithm II (NSGA-II). The experimental results show that the conflict-based strategy outperform the Kendall-based partitioning method. Additionally, the conflict-based and random partition strategies outperform NSGA-II in all the test problems considered in this study. Regarding these two partition strategies, the conflict-based partition strategy achieves a better distribution of solutions than that achieved by the random strategy. In problems in which the degree of conflict among pairs of objectives is different, the conflict-based strategy presents a better performance.

The remainder of this paper is structured in the following manner. The next section presents some basic concepts and the notation adopted throughout the paper. Section 3 briefly describes the relevant research related to our work. Section 4 introduces the new partition strategy based on conflict information. Section 5 presents an experimental analysis to evaluate the effectiveness of the new partition strategy. Finally, we present our conclusions in Section 6.

2. Basic concepts and notation

Definition 1 (*Multiobjective optimization problem*). A Multiobjective Optimization Problem (MOP) is defined as:

$$\begin{aligned} &\text{Minimize} \quad \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})]^T \\ &\text{subject to} \quad \mathbf{x} \in \mathcal{X}. \end{aligned} \tag{1}$$

The vector $\mathbf{x} \in \mathbb{R}^n$ is formed by *n decision variables* representing the quantities for which values are to be chosen in the optimization problem. The *feasible set* $\mathcal{X} \subseteq \mathbb{R}^n$ is implicitly determined by a set of equality and inequality constraints. The vector function $\mathbf{f} : \mathcal{X} \to \mathbb{R}^M$ is composed of $M \geqslant 2$ scalar *objective functions* $f_i : \mathbb{R}^n \to \mathbb{R}$ $(i = 1, \ldots, M)$. In multiobjective optimization, the sets $\mathbb{R}^n$ and $\mathbb{R}^M$ are known as *decision variable space* and *objective function space*, respectively. The image of $\mathcal{X}$ under the function $\mathbf{f}$ is a subset of the objective function space denoted by $\mathcal{Z} = \mathbf{f}(\mathcal{X})$ and referred to as the *feasible set in the objective function space*.

Definition 2 (*Objective set* Φ). The objective set of a MOP is determined by the set $\Phi = \{f_1, f_2, \ldots, f_M\}$ containing the M objective functions to be optimized.

[1] The dominance relation induced by a given set F of objectives is defined by $\preceq_F = \{(\mathbf{x}, \mathbf{y}) | \forall f_i \in F : f_i(\mathbf{x}) \leqslant f_i(\mathbf{y})\}$.