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Weighted operators based on dissimilarity function ☆

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ABSTRACT

This paper talks about weighted operators based on dissimilarity function and discusses the monotone non-decreasingness of these operators, i.e., it looks for conditions leading to aggregation operators. Moreover, the paper discusses the shift-invariance and weak monotone non-decreasingness of mentioned operators. It discusses minimization based operators $A_{\mathbf{w},D}$, $A_{\mathbf{g},D}$, where $\mathbf{w}$ is a weighting vector, $\mathbf{g}$ is a vector of weighting functions, and D is a dissimilarity function $D(x,y) = (f(x) - f(y))^2$. Following $A_{\mathbf{w},D}$ we recognize the class of arithmetic means, ordered weighted averaging *OWA* operators and their extensions. Operators $A_{\mathbf{w},D}$ are monotone non-decreasing, and hence shift-invariant and weak monotone non-decreasing, too. By the operators $A_{\mathbf{g},D}$ we introduce a generalization of the operators $A_{\mathbf{w},D}$. The operators $A_{\mathbf{g},D}$ cover the class of mixture operators, quasi-mixture operators, and their extensions. In general, these operators need not be non-decreasing, nor shift-invariant, and hence nor weak monotone non-decreasing.

1. Introduction

With the aggregation of several input values into one representative output we meet almost in all areas of life of human society, whether in the statistics evaluation in all areas of natural sciences and humanities as well as in economics sciences, or even in multicriteria decision making. In the literature exists a lot of well-known aggregation methods and ways of determination of weights (as constants) on an expression of relative importance of individual inputs, too. For more information see, for example Saaty [17], Torra and Narukawa [21], Yager [25–27]. Aggregation functions based on dissimilarities have been studied by several authors, e.g. Bustince et al. [4–6], Calvo et al. [7–9,13]. It is often necessary to measure the difference or disagreement between a set of inputs and the corresponding output. One of the possibilities is to use dissimilarity (or penalty) functions. The main idea is to give a dissimilarity function, use it as a measure of dissimilarity by finding an aggregation function that minimizes the difference between inputs and output. In other words we look for an aggregated value which minimizes the dissimilarity. Recently, many researchers study weighted operators (weighted functions), which weights are represented by weighting functions of input values. These operators are called mixture operators or even mixture functions and by certain way penalize bad properties or alternatives and reward good properties or alternatives. For more information see, for example Beliakov et al. [2], Mesiar et al. [11–13], Ribeiro and Marques Pereira [14–16], Wilkin and Beliakov [22]. Mixture operators have the big potential and flexibility. But in general, these mixture operators need not be non-decreasing nor shift-invariant, and hence nor weak monotone non-decreasing. The main property of aggregation operators,

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in other words of aggregation functions, is just non-decreasingness. This property represents very important purpose because the increase in one individual score of alternative or utility should not lead to the decrease of the overall score or utility. Some interesting aggregations of utilities can be found, for example in Špirková and Král' [19,20]. On the other side, there exist descriptive statistics which are not monotone non-decreasing (hence, they are not classified as aggregation functions), but they have very significant practical and theoretical importance. Typical examples of non-monotone averages are, for example mode, least median of squares, least trimmed squares, Gini mean and Lehmer mean, (Wilkin and Beliakov [22]). But mentioned averages are weak monotone non-decreasing. Weak monotone non-decreasingness is very useful for calculating representative values of clusters of data in the presence of outliers. According to Wilkin [23], cluster structure may change when only some inputs are increased (or decreased), but it does not change when all inputs are changed by the same value. With other typical non-monotone averaging functions which have the high importance you can meet in Angelov and Yager [1] or Wilkin et al. [24].

The main goal of this paper is to offer summary of minimization-based operators $A_{\mathbf{w},D}$ with a weighting vector $\mathbf{w}$, and $A_{\mathbf{g},D}$ with a vector of weighting functions $\mathbf{g}$, where D is a dissimilarity function $D(x,y)=(f(x)-f(y))^2$. In the case of $A_{\mathbf{w},D}$ we recognize the class of the arithmetic means, ordered weighted averaging *OWA* operators, and their extensions. These operators satisfy the condition of non-decreasingness of aggregation operators. Furthermore, we introduce minimization based operators $A_{\mathbf{g},D}$, which represent a generalization of the operators $A_{\mathbf{w},D}$. Moreover, these operators represent a generalization of mixture operators, quasi-mixture operators and their extensions. These operators need not be non-decreasing, it means that need not be aggregation operators. Moreover, these operators need not be shift-invariant and hence nor weak monotone non-decreasing. Another aim of this paper is to expand investigation of monotone and weak monotone non-decreasingness of mentioned operators.

This paper is organized as follows. In Section 2, we present the basic definitions of aggregation function, dissimilarity function and shift-invariance and weakly monotonicity. In Section 3, we express individual arithmetic means and *OWA* operators as aggregation functions $A_{\mathbf{w},D}$ based on dissimilarity function $D(x,y)=(f(x)-f(y))^2$ and the class of mixture operators, which can be expressed as dissimilarity based functions $A_{\mathbf{g},D}$. The monotone non-decreasingness of the $A_{\mathbf{g},D}$ operators can be violated, in general. We give and extend the sufficient conditions for their monotonicity, shift-invariance and weak monotonicity. Section 4 contains some conclusions and indications of further research in this domain.

2. Preliminaries

Let $I\subset R$ be a closed real non-trivial interval, $I=[a,b]$ and $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ be an input vector. The interval I is usually some of the intervals $[0,1]$ or $[0,\infty]$. Following Beliakov et al. [2], Bustince et al. [4], Calvo et al. [9], Grabisch et al. [10] and Torra and Narukawa [21] we give the next definition of aggregation function.

Definition 2.1. A function $A:[a,b]^n\to[a,b]$ is called an aggregation function if it is monotone non-decreasing in each variable and satisfies $A(\mathbf{a})=a$, $A(\mathbf{b})=b$, where $\mathbf{a}=(a,a,\ldots,a)$, $\mathbf{b}=(b,b,\ldots,b)$.

Definition 2.2. An aggregation function A is called averaging if it is bounded by the minimum and maximum of its arguments

$$min(\mathbf{x}):=min(x_1,x_2,\ldots,x_n)\leqslant A(x_1,x_2,\ldots,x_n)\leqslant max(x_1,x_2,\ldots,x_n)=:max(\mathbf{x}).$$

We recall basic definitions of a monotonicity, weak monotonicity and shift-invariant with respect to Wilkin and Beliakov [22].

Definition 2.3. A function $A:[a,b]^n\to[-\infty,\infty]$ is monotone non-decreasing if and only if, $\forall\mathbf{x},\mathbf{y}\in[a,b]^n$, $\mathbf{x}\leqslant\mathbf{y}$ then $A(\mathbf{x})\leqslant A(\mathbf{y})$.

Definition 2.4. A function $A:[a,b]^n\to[-\infty,\infty]$ is weakly monotone non-decreasing (or directionally monotone) if $A(\mathbf{x}+k\mathbf{1})\geqslant A(\mathbf{x})$ $\forall\mathbf{x}$ and for any $k>0$, $\mathbf{1}=(\underbrace{1,1,\ldots,1}_{\text{n times}})$, such that $\mathbf{x},\mathbf{x}+k\mathbf{1}\in[a,b]^n$.

Definition 2.5. A function $A:[a,b]^n\to[a,b]$ is shift–invariant (stable for translations) if $A(\mathbf{x}+k\mathbf{1})=A(\mathbf{x})+k$ whenever $\mathbf{x},\mathbf{x}+k\mathbf{1}\in[a,b]^n$ and $A(\mathbf{x})+k\in[a,b]$.

On the basis of (Bustince et al. [4–6], Calvo et al. [7], Mesiar et al. [12,13]), we recall the definition of dissimilarity function on I.

Definition 2.6. Let $K:R\to R$ be a convex function with unique minimum $K(0)=0$ and let $f:I\to R$ be a continuous strictly monotone function. Then the function $D:I^2\to R$ given by $D(x,y)=K(f(x)-f(y))$ is called a dissimilarity function (on I).

On the basis of the properties of functions K and f a dissimilarity function satisfies the following axioms: