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# A framework for state attraction of discrete event systems under partial observation

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ABSTRACT

State attraction for discrete event systems (DES) addresses the problem of reaching a desired subset of the plant state space after a bounded number of event occurrences. The problem of state attraction arises for example in fault-tolerant supervisory control or in the control of reconfigurable manufacturing systems, and is also applicable to systems biological problems such as the control of gene regulatory networks.

State attraction is investigated with the assumption of full event observation in the existing literature. This paper extends the concept of state attraction to the case of partial observation. The notion of weak attraction under partial observation (WAPO) is introduced and necessary and sufficient conditions for the existence of a supervisor under partial observation that achieves WAPO are derived. Furthermore, a solution algorithm is proposed that finds such supervisor whenever it exists. It is shown that such supervisor can always be realized as a subautomaton of the observer automaton of the DES plant. An application example from systems biology illustrates the obtained results.



## 1. Introduction

The supervisory control theory by Ramadge/Wonham [35] is a well-established theory for the control of systems that can be modeled as discrete event systems (DES). In the classical formulation, it addresses the synthesis of a *supervisor* in order to ensure that a DES *plant* fulfills a given formal language *specification*. Various approaches in the literature extend the classical formulation to supervisory control under *partial observation* [25], *decentralized* supervisory control [26,32,51], supervisory control with *hierarchical abstractions* [10,39,48], *optimal control* [41], *state attraction* [4,5], *real-time scheduling* [34], *diagnosability* [36,50] and supervisory control approaches for *fuzzy discrete event systems* [17,18,27–29].

The particular supervisory control problem studied in this paper is *state attraction* as introduced in [4,33]. It amounts to driving the state of a DES plant to a pre-specified desired subset of the state space after a bounded number of event occurrences, and staying in this subset indefinitely. If a supervisor that enforces this behavior exists, the subset is denoted as a *week attractor*. It is shown in [4] and related works such as [5,22] that, if such supervisor exists, it can be realized as a *state-feedback* supervisor on the state space of the DES plant. Moreover, the supervisor can be computed with polynomial complexity in the number of plant states. In the recent literature, state attraction is used for the fault-tolerance of DES [23,45] and for the supervisory control of reconfigurable manufacturing systems [12,40]. Moreover, the problem of state attraction is highly relevant in the field of systems biology or, more precisely, the field of modeling and controlling gene

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regulatory networks. Such networks allow for DES models, and require control towards a desired attractor [1,8,42]. For example, stem cells shall be stimulated by external controls such that they differentiate toward a desired cell state, that is a desired attractor of the underlying model [38].

It has to be noted that the current work on state attraction of DES is concerned with the case of full event observation, that is, all plant events can be observed by the supervisor. However, to the best of our knowledge, the case of state attraction under partial observation has not been studied in the literature. In this paper, we consider state attraction under partial observation, whereby the DES plant initially resides in a set of states, and it is required to use a supervisor under partial observation (SPO) in order to move the plant state to a desired set of states. As the first contribution of the paper, we introduce the notion of *weak attraction under partial observation* (WAPO) that formalizes the described setting. As the second contribution of the paper, we derive necessary and sufficient conditions for the existence of an SPO that achieves WAPO by moving the state of the DES plant from a given set of states to the set of desired states. In particular, we show that such SPO can be realized as a subautomaton of the *observer automaton* of the DES plant. As the third contribution we propose an algorithm for the computation of such state-feedback supervisor. Our algorithm is polynomial in the number of plant states and observer states, and finds a solution whenever such solution exists. Here, it has to be noted that the number of observer states can be exponential in the number of plant states in the worst case. However, this is not a shortcoming of the proposed approach but a general property of state observation under partial event observation [43]. The practicability of the concepts and results in the paper is supported by a gene regulation example from systems biology.

The related problem of *optimal control* of DES under partial observation is studied in [30]. In that work, the task is to drive the system state of a DES plant from a given initial state to a unique marked state with minimum cost. The problem solution suggests constructing a specific observer automaton, that keeps track of the cost structure of the original DES plant. Then, an optimal SPO is computed on the observer state space, whereby it is left as an open question if that SPO is also optimal for the original DES plant. Differently, the problem studied in this paper is not concerned with optimality. Furthermore, our setup is not restricted to a single initial state and a single marked state, that has to be reached after a bounded number of event occurrences but allows for a set of start states and desired states.

The remainder of the paper is organized as follows. In Section 2, we introduce the DES notation and provide basic results regarding supervisory control, partial observation and state attraction for DES. Section 3 introduces the new notion of weak attraction under partial observation (WAPO) and presents our novel results regarding supervisor existence and supervisor computation for WAPO including detailed proofs. The main results of the paper are illustrated by an application example in Section 4 and conclusions are given in Section 5.

## 2. Preliminaries

We first present basic notions of discrete event systems (DES) and supervisory control theory as in [6,49]. In addition, we recall results from supervisory control under partial observation [6], and elaborate the idea of state attraction for the case of full observation [4].

### 2.1. Discrete event systems

The set of all finite strings over a finite alphabet $\Sigma$ is denoted as $\Sigma^\star$. The concatenation of two strings $s_1,\ s_2 \in \Sigma^\star$ is written as $s_1 s_2 \in \Sigma^\star, s_1 \leqslant s$ defines that $s_1$ is a *prefix* of $s$ and $|s|$ denotes the length of $s$. The empty string is denoted as $\epsilon \in \Sigma^\star$ such that $s\epsilon = \epsilon s = s$ for all $s \in \Sigma^\star$ and $|\epsilon| = 0$. A *language* over $\Sigma$ is a subset $L \subseteq \Sigma^\star$. The *prefix closure* of $L$ is $\overline{L} := \{s_1 \in \Sigma^\star |\ \exists s \in L \text{ s.t. } s_1 \leqslant s\}$, and a language $L$ is *prefix closed* if $L = \overline{L}$.

The *natural projection* $p_i : \Sigma^\star \to \Sigma_i^\star, i = 1, 2$, for the (not necessarily disjoint) union $\Sigma = \Sigma_1 \cup \Sigma_2$ is defined iteratively: (1) let $p_i(\epsilon) := \epsilon$; (2) for $s \in \Sigma^\star, \sigma \in \Sigma$, let $p_i(s\sigma) := p_i(s)\sigma$ if $\sigma \in \Sigma_i$, or $p_i(s\sigma) := p_i(s)$ otherwise.

We model a DES as a finite state automaton $G = (X, \Sigma, \delta, x_0)$, with the finite set of *states* $X$; the finite alphabet of *events* $\Sigma$; the partial *transition function* $\delta : X \times \Sigma \to X$ and the *initial state* $x_0 \in X$.[1] We write $\delta(x,\sigma)!$ if $\delta$ is defined at $(x,\sigma)$ and extend the transition function $\delta$ to a partial function on $X \times \Sigma^\star$ in the usual way. Furthermore, we introduce the automaton $G_x = (X, \Sigma, \delta, x)$ that differs from $G$ only in the modified initial state $x$. The behavior of $G$ is characterized by its *closed language* $L(G) := \{s \in \Sigma^\star | \delta(x_0, s)!\}$. The synchronous composition $G_1 \| G_2$ of two automata $G_i = (X_i, \Sigma_i, \delta_i, x_{0,i}), i = 1, 2$, is defined in the usual way [6].

Let $G = (X, \Sigma, \delta, x_0)$ and $G' = (X', \Sigma, \delta', x'_0)$ be finite state automata. $G'$ is a *subautomaton* of $G$, denoted as $G' \sqsubseteq G$, if $X' \subseteq X, x'_0 = x_0$ and for all $x \in X'$ and $\sigma \in \Sigma$, it holds that $\delta'(x,\sigma)! \Rightarrow \delta'(x,\sigma) = \delta(x,\sigma)$ [24]. $G'$ is a *strict subautomaton* of $G$ if additionally $\delta(x,\sigma) \in X' \Rightarrow \delta'(x,\sigma) = \delta(x,\sigma)$. That is, $G'$ is a strict subautomaton of $G$ if all transitions of states in $X'$ that exist in $G$ are retained in $G'$.

### 2.2. Controllability

In a supervisory control context, we write $\Sigma = \Sigma_c \dot\cup \Sigma_u$ to distinguish *controllable* ($\Sigma_c$) and *uncontrollable* ($\Sigma_u$) events. Let $G$ be a plant automaton. A language $K \subseteq L(G)$ is said to be *controllable* for $L(G)$ and $\Sigma_u$ if $\overline{K}\Sigma_u \cap L(G) \subseteq \overline{K}$.

[1] Note that we do not include marked states in this definition since they are not relevant in the scope of this paper.