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Novel delay-dependent stability criterion for time-varying delay systems with parameter uncertainties and nonlinear perturbations

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ABSTRACT

This study proposes a novel approach to obtain less conservative conditions for the robust stability analysis for time-varying delay systems with parameter uncertainties and nonlinear perturbations. Specifically, first, we divide the time-varying delay into non-uniformly subintervals and decompose the corresponding integral intervals to estimate the bounds of integral terms more exactly. Second, novel delay-derivation-dependent stability criteria are derived by introducing a parameter γ in the Lyapunov–Krasovskii functional. Third, we define some new variables to deal with uncertain parameters. Finally, two numerical examples are presented to demonstrate the effectiveness and advantages of the theoretical results.

1. Introduction

In the past decade, investigation of time delay systems has attracted increasing attention due to the fact that time delay is inevitable in dynamical systems and often leads to poor performance or even instability [2,4–11,17,19,22,23,25,28–41]. Meanwhile, time delay systems have received considerable interest for their extensive applications in practical systems, such as mechanics, physics, economy, medicine, biology, and engineering systems. However, in practice, the stability of time delay systems may be destroyed by its uncertainties and nonlinear perturbations [3,16,20,21,24,32]. Therefore, it is necessary to study the stability analysis of time delay systems with uncertain parameters and nonlinear perturbations.

To date, some researches have investigated the stability analysis of time-invariant delay systems. For example, the stability analysis for singular systems with a constant time delay is studied in [17]. Similarly, Ref. [19,30] derive stability criteria for the time-invariant delay systems. In addition, Ref. [12] investigates the problem of fuzzy-model-based D-stability and non-fragile control for discrete-time descriptor systems with multiple constant delays. However, it is well known that the time delay is not always invariant. Time-varying delay in a given range is very common, see [2–11,13–16,18,20–25,28,29,31–34] and the references therein. Moreover, for the stability analysis of time delay systems, the target is to reduce the conservativeness. The delay partition approach [13,26,27,32] has been proved to be a less conservative method

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in time-delay systems. For instance, the study [13] investigates the stochastic stability for a class of semi-Markovian systems with mode-dependent time-variant delays by dividing the variation interval of time delay into l parts with equal length. Besides, some stability criteria for linear systems with time-varying delays are obtained in [32] by decomposing the delay interval into multiple equidistant subintervals. Furthermore, the information about the derivative of time delays should also be considered to obtain less conservative conditions. For instance, Ref. [1] investigates the control of a power system with time delay, where the derivative of time delay exists but must be less than one. Such strict constraint is removed in [18] by using some free weight matrices. Although this method seems to be effective for achieving less conservative conditions, it increases the complexity of computation. It is, therefore, an emphasis on the current study is to find a more effective approach to remove the strict constraint that the derivative of time delay must be less than one.

Motivated by the issues discussed above, the robust stability of time-varying delay systems with parameter uncertainties and nonlinear perturbations is investigated in this study. A novel method is proposed to obtain less conservative conditions. To this end, these criteria are derived via partitioning the time-varying delay into non-uniformly subintervals and decomposing the corresponding integral intervals. By improving Lyapunov–Krasovskii functional, more general linear matrix inequalities (LMIs) conditions on the stability of the delay systems are established. Specifically, on one hand, by taking $\int_{t-\delta\tau}^{t} x(s)\,ds$ and $\int_{t-\tau}^{t-\delta\tau} x(s)\,ds$ as augmented variables in the Lyapunov–Krasovskii functional, the stability criteria can utilize more information on state variables. On the other hand, by improving the term $\int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)\,ds$ as $\int_{t-\gamma\tau(t)}^{t} x^T(s)Q_1x(s)\,ds$ in the Lyapunov–Krasovskii functional, less conservative delay-derivative-dependent criteria for delay systems can be obtained via adjusting the parameter γ. Furthermore, some new variables are defined to deal with uncertain parameters. Finally, the stability criteria obtained turn out to be feasible and effective via two numerical examples.

The rest of the current paper is organized as follows. Section 2 introduces the problem formulation and some preliminaries. Section 3 and 4 present some stability criterion obtained for time delay systems. Section 5 gives some numerical examples to demonstrate the effectiveness of our main result. Finally, Section 6 draws the conclusion.

Notations: The notations used throughout the paper are fairly standard. The superscript 'T' stands for matrix transposition; R^n denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; the notation $P > 0$ means that P is a positive definite matrix; I_n and $0_{n\times n}$ represent identity matrix and zero matrix with dimension n, respectively; and diag $(\cdot)$ denotes the diagonal matrix. In symmetric block matrices, we use an asterisk $(^*)$ to represent a term which is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

Consider the time-varying delay systems with nonlinear perturbations as following [2,7,9,10,24,31,34]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + Ff(x(t),t) + Gg(x(t-\tau(t))), & t > 0 \\ x(t+\theta) = \phi(\theta), & \forall\theta \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is state vector of the systems. $A, B, F, G \in \mathbb{R}^{n\times n}$ are constant matrices. $\tau(t)$ is time-varying delay satisfying $0 \leqslant \tau(t) \leqslant \tau,\ \dot{\tau}(t) \leqslant \tau_d < \infty$. $f(x(t),t) \in R^n$ and $g(x(t-\tau(t))) \in R^n$ are unknown nonlinear perturbations with $x(t)$ and $x(t-\tau(t))$, respectively. $\phi(\cdot)$ is an initial function which is continuously differentiable on $[-\tau, 0]$.

In order to conduct the stability analysis for the aforementioned systems, it is necessary to make the following assumption and lemma:

Assumption 2.1 *[2,7,9,10,18,19,24,31,34]*. The nonlinear perturbations $f(x(t),t) \in R^n$ and $g(x(t-\tau(t))) \in R^n$ satisfy:

$$f^T(x(t),t)f(x(t),t) \leqslant \alpha^2 x^T(t)x(t) \tag{2}$$

$$g^T(x(t-\tau(t)))g(x(t-\tau(t))) \leqslant \beta^2 x^T(t-\tau(t))x(t-\tau(t)) \tag{3}$$

where α and β are two known positive constants.

Lemma 2.1 [6]. *For any positive definite matrix $M \in R^{n\times n}$, scalars $h_2 > h_1 \geqslant 0$, vector function $w : [h_1, h_2] \mapsto R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$-(h_2-h_1)\int_{t-h_2}^{t-h_1} w^T(s)Mw(s)\,ds \leqslant -\left(\int_{t-h_2}^{t-h_1} w(s)\,ds\right)^T M \left(\int_{t-h_2}^{t-h_1} w(s)\,ds\right)$$

3. Asymptotical stability analysis for time delay systems

In order to derive less conservative criteria, we firstly analyze the method of choosing Lyapunov–Krasovskii functional for the above-mentioned time-varying delay systems. Subsequently, asymptotically stable conditions for systems (1) are presented in this section.