Information Sciences xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A note on "An extension of the migrative property for uninorms" ☆

Yong Su, Wenwen Zong, Hua-Wen Liu *

School of Mathematics, Shandong University, Jinan, Shandong 250100, China

ARTICLE INFO

Article history:
Received 3 December 2013
Received in revised form 21 February 2014
Accepted 20 May 2014
Available online xxxx

Keywords:
Migrativity
Fuzzy connective
Uninorm

ABSTRACT

This paper shows by an example that **Proposition 15**(iv)(**c**) in Mas et al. (2013) about migrative property for uninorms in $\mathcal{U}_{\cos,\min}$ is false, and corrects these propositions.

1. Introduction

Mas et al. [3] introduced the concept of (α, U_0)-migrative uninroms and discussed the properties about (α, U_0)-migrative uninroms. Here, we firstly recall some definitions and results about uninorms that are continuous in the open unit square $]0,1[^2$ and then about (α, U_0)-migrative uninroms.

Definition 1.1. A binary operation $U : [0,1]^2 \to [0,1]$ is called a uninorm if it is associative, commutative, non-decreasing in each place and has a neutral element $e \in [0,1]$.

Uninorms that are continuous in the open unit square $]0,1[^2$ were characterized in [2] as follows:

Theorem 1.1 (Hu and Li [2], Ruiz and Torrens [4] for the current version). *Suppose U is a uninorm continuous in* $]0,1[^2$ *with neutral element* $e \in]0,1[$. *Then either one of the following cases is satisfied:*

(a) *There exists* $\mu \in [0,e[, \lambda \in [0,\mu]$, *two csontinuous t-norms* T_1 *and* T_2 *and a representable uninorm R such that U can be represented as*

☆ Supported by the National Natural Foundation of China (No. 61174099) and the Research Found for the Doctoral Program of Higher Education of China (No. 20120131110001).

* Corresponding author.
E-mail address: hw.liu@sdu.edu.cn (H.-W. Liu).

http://dx.doi.org/10.1016/j.ins.2014.05.038
0020-0255/© 2014 Published by Elsevier Inc.

$$U(x,y)=\begin{cases}\lambda T_1\left(\frac{x}{\lambda},\frac{y}{\lambda}\right) & \text{if } x,y\in[0,\lambda],\\ \lambda+(\mu-\lambda)T_2\left(\frac{x-\lambda}{\mu-\lambda},\frac{y-\lambda}{\mu-\lambda}\right) & \text{if } x,y\in[\lambda,\mu],\\ \mu+(1-\mu)R\left(\frac{x-\mu}{1-\mu},\frac{y-\mu}{1-\mu}\right) & \text{if } x,y\in]\mu,1[,\\ 1 & \text{if } \min(x,y)\in]\lambda,1] \text{ and } \max(x,y)=1,\\ 1 \text{ or } \lambda & \text{if } (x,y)\in\{(\lambda,1),(1,\lambda)\},\\ \min(x,y) & \text{otherwise.}\end{cases} \tag{1}$$

(b) *There exists $\nu\in]e,1],\omega\in[\nu,1]$, two continuous t-conorms S_1 and S_2 and a representable uninorm R such that U can be represented as*

$$U(x,y)=\begin{cases}\nu+(\omega-\nu)S_1\left(\frac{x-\nu}{\omega-\nu},\frac{y-\nu}{\omega-\nu}\right) & \text{if } x,y\in[\nu,\omega],\\ \omega+(1-\omega)S_2\left(\frac{x-\omega}{1-\omega},\frac{y-\omega}{1-\omega}\right) & \text{if } x,y\in[\omega,1],\\ \nu R\left(\frac{x}{\nu},\frac{y}{\nu}\right) & \text{if } x,y\in]0,\nu[,\\ 0 & \text{if } \max(x,y)\in[0,\omega[\text{ and } \min(x,y)=0,\\ 0 \text{ or } \omega & \text{if } (x,y)\in\{(\omega,0),(0,\omega)\},\\ \max(x,y) & \text{otherwise.}\end{cases} \tag{2}$$

The class of all uninorms that are continuous in $]0,1[^2$ will be denoted by $\mathcal{U}_{\cos}$. A uninorm as in (1) will be denoted by $U\equiv\langle T_1,\lambda,T_2,\mu,(R,e)\rangle_{\text{icos,min}}$ and the class of all uninorms that are continuous in the open unit square of this form will be denoted by $\mathcal{U}_{\cos,\min}$. Analogously, a uninorm as in (2) will be denoted by $U\equiv\langle S_1,\nu,S_2,\omega,(R,e)\rangle_{\cos,\max}$ and the class of all uninorms that are continuous in the open unit square of this form will be denoted by $\mathcal{U}_{\cos,\max}$.

Definition 1.2 (Mas et al. [3]). Consider $\alpha\in[0,1]$ and let U_0,U_1 be two uninorms with neutral element $e\in]0,1[$. We will say that a uninorm U is (α,U_0)-migrative or that U is α-migrative over U_0 if

$$U(U_0(\alpha,x),y)=U(x,U_0(\alpha,y))\quad\text{for all}\quad x,y\in[0,1]. \tag{3}$$

Proposition 1.1 (Mas et al. [3]). *Consider $\alpha\in[0,1]$ and let U_0,U be two uninorms with neutral element $e\in]0,1[$. Then a uninorm U is (α,U_0)-migrative if and only if*

$$U_0(\alpha,x)=U(\alpha,x)\quad\text{for all } x\in[0,1]. \tag{4}$$

2. A counterexample

To illustrate that **Proposition 15** (iv)(**c**) in [3] about migrative property for uninorms in $\mathcal{U}_{\cos,\min}$ is false, we construct two non-representable uninorms $U_0\equiv\langle T_0',\lambda_0,T_0'',\mu_0,(R_0,e)\rangle_{\cos,\min}$ and $U\equiv\langle T',\lambda,T'',\mu,(R,e)\rangle_{\cos,\min}$ in $\mathcal{U}_{\cos,\min}$ with neutral element e, where $\mu<\mu_0<\alpha$, R is $(\frac{\alpha-\mu}{1-\mu},R_0)$-migrative and U is not (α,U_0)-migrative.

Proposition 15 (iv)(c) in [3] is as follows:

Let $\alpha\in[0,1]\setminus\{e\}, U_0\equiv\langle T_0,e,S_0\rangle\equiv\langle T_0',\lambda_0,T_0'',\mu_0,(R_0,e)\rangle_{\cos,\min}$ be a non-representable uninorm in $\mathcal{U}_{\cos,\min}$ with neutral element e, and $U\equiv\langle T',\lambda,T'',\mu,(R,e)\rangle_{\cos,\min}$ a non-representable uninorm with neutral element e. If $\alpha\geqslant\max(\mu,\mu_0)$, then U is (α,U_0)-migrative if and only if R is $(\frac{\alpha-\mu}{1-\mu},R_0)$-migrative.

Example 2.1. Let $\alpha=\frac{13}{16}, e=\frac{1}{2}, \mu=\frac{1}{4}, \mu_0=\frac{1}{3}$.

The 3Π operator is defined as

$$R_0(x,y)=\begin{cases}0 & \text{if } (x,y)\in\{(1,0),(0,1)\},\\ \frac{xy}{(1-x)(1-y)+xy} & \text{otherwise.}\end{cases}$$

It is easily verified that the uninorm R_0 can be written in the following form [1]:

$$R_0(x,y)=h_0^{-1}(h_0(x)+h_0(y))$$

with the help of the continuous and strictly increasing function $h_0:[0,1]\to[-\infty,+\infty]$ defined by $h_0(x)=\log\left(\frac{x}{1-x}\right)$. Moreover, $h_0(0.5)=0$, and $R_0(0.5,x)=x$ for any $x\in[0,1]$, whence the neutral element of R_0 is 0.5. Routine calculation shows that $h_0^{-1}(x)=\frac{e^x}{1+e^x}$ for all $x\in[-\infty,+\infty]$.

Now we define a representable uninorm R which is $(\frac{\alpha-\mu}{1-\mu},R_0)$-migrative.

Q1 Please cite this article in press as: Y. Su et al., A note on "An extension of the migrative property for uninorms", Inform. Sci. (2014), http://dx.doi.org/10.1016/j.ins.2014.05.038