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An optimal machine maintenance problem with probabilistic state constraints

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ABSTRACT

We consider a machine that is maintained via two types of maintenance action: (i) continuous (minor) maintenance that curbs natural degradation of the machine; and (ii) overhaul (major) maintenance that takes place at certain discrete time points and significantly improves the condition of the machine. We introduce an impulsive stochastic differential equation to model the condition of the machine over the time horizon. The problem we investigate is to choose the continuous maintenance rate and the overhaul maintenance times to minimize the total cost of operating and maintaining the machine, where probabilistic state constraints are imposed to ensure that the machine's state and output meet minimum acceptable levels with high probability. This impulsive stochastic optimal control problem is first transformed into a deterministic optimal control problem with state jumps and continuous inequality constraints. We then show that this equivalent problem can be solved using a combination of the control parameterization technique, the time-scaling transformation, and the constraint transcription method. Finally, we illustrate our approach by solving a numerical example.

1. Introduction

In any industrial setting, maintenance is paramount to ensuring reliable machine operation. Although maintenance operations are sometimes costly and onerous, ignoring maintenance will increase the likelihood of machine failure, thus potentially leading to major disruptions in production at some future time. Hence, effective maintenance policies are essential for production planning purposes.

Maintenance policies in the literature can be classified into three types: (i) failure-based, where the maintenance is performed after failure of the machine; (ii) time-based, where maintenance is scheduled at fixed times; and (iii) preventive-based, where maintenance is scheduled depending on the machine's state. An importance part of preventive maintenance is the modeling of the machine's deterioration process. The condition-based maintenance approach that we propose in this paper models the state of the machine using a stochastic process, which allows for the random noise and disturbances present in any real-life system. Maintenance overhauls are scheduled so that there is a sufficiently high probability that the machine's state will always be above a minimum acceptable level.

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Previous research on machine maintenance has typically studied the performance of different maintenance and production policies separately, even though these two activities are intrinsically linked [5,21]. However, in recent years, more researchers have begun to design production and maintenance policies simultaneously [4,22,16]. Graves and Lee [6] considered the simultaneous optimization of production together with preventive maintenance and scheduling decisions, but their model allows just one maintenance activity during the planning horizon. Similarly, the maintenance scheduling model studied by Hsu et al. [8] also allowed for just one maintenance activity throughout the planning horizon. Qi et al. [17] allowed for multiple maintenance activities, but neglected the downside risk of not performing maintenance. Cassady and Kutanoglu [3] improved on the studies in Graves and Lee [6], Qi et al. [17] by explicitly incorporating the risk of not performing maintenance. However, they assumed that the duration between maintenance times is constant, and that preventive maintenance restores the machine to an 'as good as new' condition, which is not always a realistic assumption.

Batun and Azizoğlu [2] considered a model involving multiple maintenance activities of known start times and durations, where several non-resumable production jobs need to be scheduled optimally. Sbihi and Varnier [18] improved on this model by relaxing the assumption of fixed maintenance time intervals, and imposing limits on the machine's maximum continuous working time. Pan et al. [16] considered machine degradation and variable maintenance times. However, as with Cassady and Kutanoglu [3], they assumed that preventive maintenance activities are able to restore the machine to an 'as good as new' condition. Fitouhi and Nourelfath [5] make the same assumption, but allowed for the machine's failure rate to increase with time. Wang [21], too, allowed for a time-dependent failure rate by using a Weibull distribution to model the time to failure of the machine.

This paper improves on existing models by allowing flexible maintenance time intervals while ensuring the probability of machine failure is below a minimum specified value. The optimal interval lengths between maintenance overhauls are decision variables, which must be chosen to ensure that the likelihood of machine failure is small and that a given production target is met. We formulate this problem as a special type of stochastic impulsive optimal control problem, where the state impulses are due to overhaul maintenance activities occurring at a set of discrete time points.

Impulsive optimal control problems arise in many applications [12]. Li et al. [10] present an impulsive optimal control model for designing optimal trajectories of horizontal oil wells. Loxton et al. [14] also apply impulsive optimal control methodologies to determine the optimal switching instants for a switched-capacitor DC/DC power converter. In another paper, Wu and Teo [23] consider the optimization of a general impulsive system that can be used as a model for many real-life systems such as robots, locomotives, hybrid power generators and biochemical reactors. However, none of these papers consider impulsive optimal control models for machine maintenance scheduling. This paper represents the first attempt at applying optimal control techniques for impulsive systems in the machine maintenance area.

The remainder of the paper is organized as follows. In Section 2, we formulate the maintenance scheduling problem as an impulsive optimal control problem with stochastic disturbances. Then, in Section 3, we discuss a transformation technique for converting the stochastic problem into a deterministic problem. In Sections 4 and 5, we apply the time-scaling transformation and the constraint transcription technique to solve this deterministic problem. A numerical example is provided in Section 6. Section 7 concludes the paper.

2. Problem formulation

Let $x(t)$ denote the state of the machine at time t, and let $y(t)$ denote the total output produced by the machine up to time t. The machine's state and output are governed by the following system of stochastic differential equations:

$$dx(t) = (u(t) - k_1)x(t)dt + k_2 dw(t), \tag{1}$$
$$dy(t) = k_3 x(t)dt, \tag{2}$$

where $u(t)$ denotes the continuous maintenance rate; $w(t)$ denotes the standard Brownian motion with mean 0 and covariance given by

$$\mathrm{Cov}\{w(t_1), w(t_2)\} = \min\{t_1, t_2\}; \tag{3}$$

and $k_1,\ k_2$ and k_3 are given constants representing, respectively, the machine's natural degradation rate, the propensity for random fluctuations in the machine's condition, and the extent to which the machine's state influences production. We impose the following bound constraints on the continuous maintenance rate:

$$0 \leqslant u(t) \leqslant ak_1, \quad t \geqslant 0, \tag{4}$$

where $a \in (0,1)$ is a given constant.

The initial state of the machine and the initial production level are given by

$$x(0) = x^* + \delta_0, \tag{5}$$
$$y(0) = 0, \tag{6}$$

where δ_0 is a normal random variable with mean 0 and variance k_4. Note that $x(t) \approx x^*$ indicates that the machine is operating in an almost perfect condition.