Information Sciences 281 (2014) 418–442

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

An interval type-2 fuzzy model of computing with words

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ARTICLE INFO

Article history:
Received 12 May 2013
Received in revised form 20 April 2014
Accepted 26 May 2014
Available online 4 June 2014

Keywords:
Computing with word
Interval type-2 fuzzy set
Finite automata
Interval type-2 fuzzy automata
Pushdown automata

ABSTRACT

As a methodology, computing with words allows the use of words, instead of numbers or symbols, in the process of computing and reasoning and thus conforms more to humans' inference when it is used to describe real-world problems. Motivated by Zadeh's paradigm of computing with words, the literature has proposed a kind of type-1 fuzzy automata as a formal model of computing with words, which takes type-1 fuzzy subsets of symbols as input. However, type-1 fuzzy representation provides a limited platform for approximating the meaning of words since it is not able to capture linguistic uncertainty. In this paper, we develop a formal interval type-2 fuzzy model of computing with words by generalizing the existing type-1 fuzzy sets-based model to an interval type-2 fuzzy environment. Concretely, we take interval type-2 fuzzy automata (i.e., interval type-2 fuzzy finite automata and interval type-2 fuzzy pushdown automata), which combine interval type-2 fuzzy set theory and automaton theory, as a computational model of computing with words. Furthermore, we develop the extension principles to extend from computing with values to computing with words and show that computing with words can be implemented with computing with values in the interval type-2 fuzzy setting at an extra computational cost.

1. Introduction

When describing real-world problems, numerical precision may be costly and insubstantial because human thought works not at the numeric level of precision but rather at a more abstract level [4,41]. In order to abstract and formalize the reasoning process with natural languages and to enable a human-centric view in such a process, the concept of computing with words (CW) has been proposed [27,43–45,52,53,60–64,67]. As pointed out by Zadeh [60], CW combines natural language processing with computation with fuzzy variables. As a basic methodology, CW has found successful applications in information processing, decision, and control [10,15,16,26,28,42,69] ever since its introduction.

Traditionally, computing focuses on manipulation of numbers and symbols and is often described by a dynamic model that takes input in a certain format. In CW, the objects of computation come from words and propositions drawn from a natural language [5]. It is worth noting that the word "computing" in the literatures on CW often refers to modeling and reasoning methods with certain computational efficiency, instead of any formal theory of computing [4]. The lack of formal theoretical foundation has impeded the development of CW as a fundamental methodology. To fill this void in CW, Ying [59] proposed to incorporate the classical computation models into CW and suggested using fuzzy automata as a formal model for CW. More concretely, in [59], two types of fuzzy automata have been proposed, and linguistic terms are modeled as fuzzy subsets of a set of symbols and accepted as the input of the fuzzy automata with a certain degree between 0 and 1. In

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http://dx.doi.org/10.1016/j.ins.2014.05.055
0020-0255/

addition, the extension principle from computing with values to computing with words has been developed in [59]. As a step further in this direction, Wang and Qiu [50] extended the notion of CW through incorporating Turing Machine (TM) models into CW. They have studied CW under the lens of context-free grammars and regular grammars, and certain equivalence between fuzzy pushdown automata and fuzzy context-free grammars has been established along with the equivalence between fuzzy finite-state automata and regular grammars. The model of CW proposed by Cao et al. [5] is characterized by the fuzzy transition function that can be specified arbitrarily and the principles to extend from computing with words to computing with all words for the purpose of coping with fuzzy inputs. In the direction of developing a concurrency model for CW, Cao and Chen [4] proposed to incorporate Petri nets, a well-known model for concurrent computing, with fuzzy set theory. By using the methodology of fuzzy reasoning, the fuzzy Petri nets for CW proposed by Cao and Chen [4] allow computing with more words.

It is worth noting that all the formal models of CW mentioned previously are based upon ordinary fuzzy set theory (i.e., type-1 fuzzy set theory) [65]. In fact, CW using type-1 fuzzy sets has been studied in many literatures (e.g., [4,5,15,20,26,31,32,50,51,59,60]). However, the limitations of using type-1 fuzzy sets in CW have also been pointed out by some researchers [14,32,49,54]. More concretely, Herrera and Herrera-Viedma [14] pointed out that it is difficult for all individuals to agree on the same membership function (i.e., type-1 fuzzy sets) associated with linguistic terms. Turksen [49] also pointed out that type-1 fuzzy representation simplifies the approximation of the meaning representation of words by ignoring the uncertainty embedded in the spread of membership values. Given that the same word can have different meanings to different people and type-1 fuzzy sets do not provide the capability to capture such uncertainties, it has been suggested that a type-2 fuzzy set model be used in CW [3,49]. For example, Mendel [31–33] asserted that an interval type-2 fuzzy set (rather than a type-1 fuzzy set) should be used as a fuzzy set model of a word in CW.

In fact, there have been research works on model words using interval type-2 fuzzy sets (i.e., Interval Type-2 Fuzzy Sets, IT2 FSs) [24,40]. The main difference between type-1 fuzzy sets and type-2 fuzzy sets (i.e., General Type-2 Fuzzy Sets, GT2 FSs) is that the memberships of a type-1 fuzzy set are crisp numbers, whereas the memberships of a GT2 FS are type-1 fuzzy sets; hence, a GT2 FS can model more uncertainties. So far, the most widely used GT2 FSs are IT2 FSs [18,22,38,39,54,55]. Consequently, in this paper, IT2 FSs are used to model words in order to build more generally computational models for CW.

The purpose of this paper is to develop a formal interval type-2 fuzzy model of CW by exploiting IT2 FSs. More concretely, we generalize the formal model of CW proposed by Ying [59] into interval type-2 fuzzy environments. That is, we propose interval type-2 fuzzy automata, which combine interval type-2 fuzzy set theory and automaton theory, as a computational model of CW. To the best of our knowledge, little effort has been made to study (interval) type-2 fuzzy automata, although (interval) type-2 fuzzy set theory has demonstrated good performance in a number of applications. Following the formal model of CW in [59], we will present a new type of interval type-2 fuzzy automata whose inputs are strings of interval type-2 fuzzy subsets of the input alphabet. These new interval type-2 fuzzy automata can serve as formal models of computing with words. It is well-known that the two most important and simplest types of automata are finite automata and pushdown automata; they recognize regular languages and context-free languages, respectively. The behaviors of the two types of automata with words (i.e., strings of IT2 FSs) as their inputs are studied in detail in this paper.

The remainder of this paper is organized as follows. The next section briefly reviews some of the background on IT2 FSs. In Section 3, we propose a formal model of CW by exploiting interval type-2 fuzzy finite automata. In Section 4, we provide a formal model of CW by using interval type-2 fuzzy pushdown automata. Section 5 discusses some extensions of the formal model of CW. Finally, in Section 6, we draw our conclusions and present some topics for future research. The proofs of our lemmas and theorems are given in Appendix A.

2. Interval type-2 fuzzy sets

In this section, the preliminaries of IT2 FSs are briefly reviewed and new notions are proposed. See especially [36–38] for further details and properties of IT2 FSs.

The notion of a GT2 FS was introduced by Zadeh [66] as a generalization of the concept of an ordinary fuzzy set (i.e., type-1 fuzzy set). The difference between type-1 and type-2 fuzzy sets lies in the membership function. While the membership function of a type-1 fuzzy set is an accurate function, the membership grades of a GT2 FS by themselves are type-1 fuzzy sets. In CW, since the same words can have different meanings to different people, it is difficult to determine an accurate membership function for a fuzzy set, and thus GT2 FSs are more useful to incorporate linguistic uncertainties [19]. Recently, there has been a growing interest in type-2 fuzzy set and system theory [1,2,6–8,17,30,34,35]. To date, IT2 FSs are the most widely used GT2 FSs [56], and have been used successfully for decision making [9,48,54], survey processing [36], speech recognition [68], clustering [46], Web shopping [11], control [12,23,29], attribute reduction [58], ontology [21], linguistic summarization [57], field programmable gate array (FPGA) [25,47], mechanical systems [7], and computing with words [24,40]. The widely accepted definition for GT2 FSs, provided by Mendel [36], is given in Definition 1.

Definition 1 [36]. Let X be a universe of discourse. A GT2 FS $\widetilde{A}$ in X is characterized by a type-2 membership function. $\mu_{\widetilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$, i.e.,