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Identification of transparent, compact, accurate and reliable linguistic fuzzy models

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ABSTRACT

Transparency, accuracy, compactness and reliability all appear to be vital (even though somewhat contradictory) requirements when it comes down to linguistic fuzzy modeling. This paper presents a methodology for simultaneous optimization of these criteria by chaining previously published various algorithms – a heuristic fully automated identification algorithm that is able to extract sufficiently accurate, yet reliable and transparent models from data and two algorithms for subsequent simplification of the model that are able to reduce the number of output parameters as well as the number of fuzzy rules with only a marginal negative effect to the accuracy of the model.

1. Introduction

The research on fuzzy systems (see e.g. [7,9,10,23]) of last years has adequately pointed out the uniqueness and value of interpretability and has also provided means and tools for facilitation and exploitation of this property. It seems that a tentative consensus has been reached in what comprises interpretability. Aside from low-level interpretability requirements (normality, coverage, convexity and distinguishability of fuzzy partitions) that have progressively become a norm in fuzzy community, higher-level interpretability has become somewhat interchangeable with complexity (often termed as readability in interpretability context). For example, a recent work [3] considers a small number of fuzzy rules and compact (incomplete) rules for large systems instrumental to interpretability and to reflect that, the proposed hierarchical fuzzy system for assessing interpretability in this paper combines different complexity measures to produce the interpretability index.

Aside from being a measure of evaluation, interpretability index can serve as the optimization criterion for evolutionary algorithms to improve interpretability of a fuzzy system and indeed, evolutionary algorithms have become increasingly popular in fuzzy optimization [2,7,10,13,17,40]. However, these algorithms work with a family of potential solutions, are therefore computationally expensive and require many (sometimes thousands) iterations to converge. This is often unacceptable for practical applications and computationally more affordable alternatives must be sought.

Interestingly enough, most latest interpretability-related developments [2,3,17,21,24] have taken place in the context of classification where the task of a fuzzy rule-based classifier is just to assign a class label (the number of which is limited) to the sample presented to it. In modeling and control, however, the output is generally continuous imposing perhaps higher accuracy requirements and rule interpolation obtains a central place. In consequence, complexity/readability issue that is prominent in most interpretability studies becomes less important concern (note that because of the curse of dimensionality fuzzy modeling is rarely performed for large-scale systems), however, this is more than compensated by increased interpolation-driven interpretability (and other) concerns.

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The latter is the main reason why in fuzzy modeling and control we prefer to handle interpretability in a wider context where interpretability is perceived as a measure of fuzzy system consistency [34] – an umbrella term that has been coined to embrace all aspects of fuzzy system applicability in modeling (not to be confused with rule consistency utilized e.g. in [3]) – and more specifically, a measure of internal consistency (that has its own aspects of transparency, linguistic integrity and complexity).

What really unites all aspects of internal consistency is that they can be generally validated without external information (e.g. validation data). Aside from purely academic research we, however, usually want to exploit interpretability for the problem at hand and therefore an internally totally consistent fuzzy system is generally not really useful if it is numerically grossly inaccurate or its rules cannot be relied on because they express information that cannot be confirmed otherwise (by available numerical data or expert opinion). These concerns – accuracy and reliability – are the most important aspects of external consistency and, incidentally, what we typically aim for is a certain balance between internal and external consistency of the system (this is perhaps better known as interpretability-accuracy tradeoff).

In this paper our goal is to provide a new methodology that is able to handle adequately all aspects of system consistency (both internal and external) in fuzzy modeling at a moderate computational cost. For this we employ different algorithms.

The first step of the procedure is the identification of a transparent fuzzy model using the training data and a fully automatic algorithm (developed to perfection in [36] to cope with noisy environment) that has built-in mechanisms for transparency protection and reliability preservation.

The class of systems under consideration here are the fuzzy singleton (or 0th order Takagi–Sugeno) systems. What makes these systems special is that they have all the attractive properties of linguistic (Mamdani) systems, whereas numerically they are very easy to manipulate (their inference function is analytical and inexpensive) and interpolation in such systems is very intuitive.

The assessment of complexity/readability of rules is carried out in subsequent manipulation of the identified model by two further algorithms and is twofold. First, the issue of abundance of output singletons, characteristic to 0th order TS systems and the direct result of the application of the modeling algorithm in previous step, is addressed using a recently developed reduction algorithm [37]. This heavily reduces the number of output parameters and makes evident otherwise hidden redundancy of fuzzy rules that can be removed by yet another recent method [35,38].

Numerous examples (including the applications of gas furnace and acidogenic state modeling) positively confirm that what we have here is an efficient tool for minimizing the gap between accuracy (from one side) and the properties of transparency, reliability and complexity from another side.

2. Preliminaries

Consider a multi-input single-output fuzzy system, consisting of R rules:

$$\begin{aligned}&\text{IF } x_1 \text{ is } A_{1r} \text{ AND } x_2 \text{ is } A_{2r} \text{ AND} \ldots\\&\ldots \text{AND } x_N \text{ is } A_{Nr} \text{ THEN } y \text{ is } b_r\\&\text{OR} \ldots,\end{aligned} \tag{1}$$

where A_{ir} denote the linguistic labels of the ith ($i = 1,\ldots,N$) input variable (into which these variables have been partitioned) associated with the rth ($r = 1,\ldots,R$) rule, and b_r is the scalar (fuzzy singleton), associated with the rth rule.

Each A_{ir} has its representation in the numerical domain – the membership function μ_{ir} (MF). In a normal fuzzy system the number of MFs per ith variable (S_i) is relatively small – in any way, this number is rarely equal to R as the notation style in (1) implies – moreover, it is often desired that all possible unique combinations of input MFs are represented ($R = \prod_{i=1}^{N} S_i$). MFs of the system are thus shared between the rules and a separate $R \times N$ dimensional matrix that accommodates the identifiers $m_{ri} \in \{1,2,\ldots,S_i\}$ maps the existing MFs μ_i^s to the rule slots. The number of independent output singletons (T) in fuzzy singleton (0th order Takagi–Sugeno systems), on the other hand, is generally equal to R (and thus matches the notation style in (1)).

In current approach MFs μ_i^s are defined by

$$\mu_i^s(x_i) = \begin{cases} \frac{x_i - a_i^{s-1}}{a_i^s - a_i^{s-1}}, & a_i^{s-1} < x_i < a_i^s, \\ \frac{a_i^{s+1} - x_i}{a_i^{s+1} - a_i^s}, & a_i^s < x_i < a_i^{s+1}, \\ 0, & a_i^{s+1} \leqslant x_i \leqslant a_i^{s+1}, \end{cases} \tag{2}$$

by what

$$\sum_{s=1}^{S_i} \mu_i^s(x_i(k)) = 1. \tag{3}$$

The latter has become known as Ruspini [39], strong [13] or *standard* partition and is often exploited for its simplicity and for built-in low-level interpretability requirements (coverage, normality, convexity, distinguishability).