Information Sciences 278 (2014) 67–75

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Interactive fuzzy stochastic two-level linear programming with simple recourse

Masatoshi Sakawa *, Hideki Katagiri, Takeshi Matsui

Department of System Cybernetics, Faculty of Engineering, Hiroshima University, Higashi-Hiroshima 739-8527, Japan

ARTICLE INFO

Article history:
Received 28 September 2009
Received in revised form 26 April 2010
Accepted 8 March 2014
Available online 17 March 2014

Keywords:
Two-level linear programming
Random variable
Simple recourse
Interactive fuzzy programming

ABSTRACT

In this paper, assuming cooperative behavior of the decision makers, two-level linear programming problems involving random variables in constraints are considered. Using the concept of simple recourse, the formulated stochastic two-level simple recourse problems are transformed into deterministic two-level programming ones. Taking into account vagueness of judgments of the decision makers, interactive fuzzy programming is presented. In the proposed interactive method, after determining the fuzzy goals of the decision makers at both levels, a satisfactory solution is derived efficiently by updating the satisfactory degree of the decision maker at the upper level with considerations of overall satisfactory balance between both levels. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

1. Introduction

In actual decision making situations, we must often make a decision on the basis of vague information or uncertain data. For such decision making problems involving uncertainty, there exist two typical approaches: probability-theoretic approach and fuzzy-theoretic one. Stochastic programming, as an optimization method based on the probability theory, have been developed in various ways [47,48,6,27], including two-stage programming [2,3,9,10,12,20,49,50] and chance constrained programming [7,8,27,47]. Fuzzy mathematical programming representing the vagueness in decision making situations by fuzzy concepts have been studied by many researchers [22,23]. Fuzzy multiobjective linear programming, first proposed by Zimmermann [51], have been also developed by numerous researchers, and an increasing number of successful applications has been appearing [14,18,23–25,40,45,46,52]. In particular, after reformulating stochastic multiobjective linear programming problems using several models for chance constrained programming, Sakawa et al. [26,28,29] presented an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker (DM) as a generalization of their previous results [23,37–40].

However, decision making problems in decentralized organizations are often formulated as two-level programming problems with a DM at the upper level (DM1) and another DM at the lower level (DM2) [32]. Under the assumption that these DMs do not have motivation to cooperate mutually, the Stackelberg solution [5,19,42,43] is adopted as a reasonable solution for the situation. On the other hand, in the case of a project selection problem in the administrative office of a company and its autonomous divisions, the situation that these DMs can cooperate with each other seems to be natural rather than the noncooperative situation. Assuming that the DMs essentially cooperate with each other, Lai [13] and Shih et al. [41]

* Corresponding author.
E-mail address: sakawa@hiroshima-u.ac.jp (M. Sakawa).

http://dx.doi.org/10.1016/j.ins.2014.03.020
0020-0255/

proposed solution concepts for two-level linear programming problems. In their methods, the DMs identify membership functions of the fuzzy goals for their objective functions, and in particular, the DM at the upper level also specifies those of the fuzzy goals for the decision variables. The DM at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the DM at the upper level. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables. In order to overcome the problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakawa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for the DMs [33,34]. The subsequent works on two-level or multi-level programming under fuzziness have been developed [1,15,21,30,31,35,36,44].

Realizing the importance of considering not only the fuzziness but also the randomness of coefficients of objective functions or constraints in mathematical programming, some researchers developed two-stage or multi-stage fuzzy stochastic programming [11,16,17]. However, there is no study which focuses on the simultaneous consideration of two-level decision making situations and fuzzy stochastic programming approaches.

Under these circumstances, we propose a novel fuzzy stochastic two-level programming model which incorporates interactive two-level fuzzy programming into two-stage stochastic programming. In two-level programming under a cooperative relationship between the two DMs, the upper-level DM needs to select a solution that takes a balance between his/her own objective function value and the lower-level DM's objective function value. In addition, it is significant to properly represent the imprecision of the satisfaction of DMs with respect to the goals of objective function values. From these viewpoints, in the proposed interactive method, after determining the fuzzy goals of the DMs at both levels, a satisfactory solution is derived efficiently by updating the satisfactory degree of the DM at the upper level with considerations of overall satisfactory balance between the both level DMs. The proposed method has an advantage that the problem for deriving a satisfactory solution can be strictly solved by some convex programming techniques like the sequential quadratic programming method. A numerical example of two-level production planning problems is provided to illustrate the feasibility and efficiency of the proposed method.

2. Stochastic two-level linear programming problems

In this paper, we deal with two-level linear programming problems involving random variables in the right-hand side of constraints formulated as:

$$\left.\begin{array}{ll}\underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1,\boldsymbol{x}_2)=\boldsymbol{c}_{11}\boldsymbol{x}_1+\boldsymbol{c}_{12}\boldsymbol{x}_2\\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1,\boldsymbol{x}_2)=\boldsymbol{c}_{21}\boldsymbol{x}_1+\boldsymbol{c}_{22}\boldsymbol{x}_2\\ \text{subject to} & A_1\boldsymbol{x}_1+A_2\boldsymbol{x}_2=\bar{\boldsymbol{b}}\\ & \boldsymbol{x}_1\geqslant\boldsymbol{0},\boldsymbol{x}_2\geqslant\boldsymbol{0}\end{array}\right\}. \tag{1}$$

In this formulation, $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the decision maker at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the decision maker at the lower level (DM2), $\boldsymbol{c}_{11}$ and $\boldsymbol{c}_{21}$ are n_1 dimensional coefficient row vectors, $\boldsymbol{c}_{12}$ and $\boldsymbol{c}_{22}$ are n_2 dimensional coefficient row vectors, A_1 is an $m\times n_1$ coefficient matrix, A_2 is an $m\times n_2$ coefficient matrix, $z_1(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM1, $z_2(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the objective function for DM2, and $\bar{\boldsymbol{b}}$ is an m dimensional random variable column vector whose elements are independent of each other.

It is significant to note here that we are often faced with optimization problems involving randomness like (1). For instance, consider an upper level company (DM1) in charge of upper processes of n_1 types and a cooperating lower level company (DM2) in charge of lower processes of n_2 types in the production of m products. Then, there may exist a two-level optimization problem where each of DMs wants to minimize its own objective function under the situation that for each decision variable vector $\boldsymbol{x}_l$ representing the production level of DMl, the unit production cost coefficient vector for upper processes of DM1, $\boldsymbol{c}_{11}$, that for lower processes of DM1, $\boldsymbol{c}_{12}$, that for upper processes of DM2, $\boldsymbol{c}_{21}$, that for lower processes of DM1, $\boldsymbol{c}_{22}$, the unit production amount coefficient vector for upper processes of the ith product, $\boldsymbol{a}_{i1}$, and that for lower processes of the ith product, $\boldsymbol{a}_{i2}$, are known while each demand for the ith product, $b_i, i=1,2,\ldots,m$ varies randomly.

When chance constrained programming approaches [7,8,27,47] are taken to deal with mathematical programming problems with random variables, it is implicitly assumed that the realized values of the random variable coefficients cannot be observed at all until the decision is made. However, in real-world decision making problems, we are often faced with situations where the realized values of the random variables are gradually observed; firstly a DM must make a decision before he/she knows the realized values of random variables, and secondly, the penalty of violation of constraints is incorporated to compensate the violation. Such a decision making methodology is called the two-stage program, which was originally introduced by Beale [3] as a simple recourse model, and extended as more generalized recourse models [49,50] including the fixed recourse model and the complete recourse model.

Since the simple recourse model is the most fundamental and practical among recourse models in the sense that the shortage or surplus of products can be directly compensated by the purchase of equivalent alternative products or the disposal of products, in this paper, we adopt the simple recourse model together with the consideration of the imprecise nature of DM's judgment for the goals of objective function values.