Information Sciences 278 (2014) 555–558

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Correction and improvement on several results in quantitative logic ☆

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ARTICLE INFO

Article history:
Received 23 August 2009
Received in revised form 16 March 2010
Accepted 8 March 2014
Available online 3 April 2014

Keywords:
Degree of similarity
Pseudo-metric
Quantitative logic

ABSTRACT

The aim of this paper is to correct and improve some results obtained in the paper "Quantitative logic" [Information Sciences 179 (2009) 226–247].

1. Introduction

In [1], the authors introduced the concepts of truth degree of a formula, similarity degree and pseudo-metric between formulas, divergence degree and consistency degree of a theory, and hence provided a possible framework for graded approximate reasoning. However, several results in Theorem 8 and Theorem 9 in [1] are incorrect. So, in this note, we will correct them and give the detailed proof processes.

The above mentioned results are related to n-valued Łukasiewicz propositional logic system $Ł_n$, n-valued R_0-type propositional logic system $\mathcal{L}_n^*$ and fuzzy R_0-type propositional logic system $\mathcal{L}^*$. For the convenience of reading, we will use the same notations as in [1,2].

2. Corrections to results in systems $_n$ and $\mathcal{L}_n^*$

Definition 2.1 (*Wang and Zhou [1]*). Let $A = A(p_1, \dots, p_m)$ be a formula in $F(S)$ containing m atomic formulas $p_1, \dots, p_m$, and let $\overline{A}(x_1, \dots, x_m)$ be the truth function induced by A. Define

$$\tau_n(A) = \frac{1}{n^m}\sum_{i=1}^{n-1}\frac{i}{n-1}\left|\overline{A}^{-1}\left(\frac{i}{n-1}\right)\right|,$$

☆ Supported by the National Natural Science Foundation of China (Nos. 60774100, 10771129 and 61174099), the Natural Science Foundation of Shandong Province of China (No. Y2007A15) and the Specialized Research Fund for the Doctoral Program of Higher Education of China (No. 20120131110001).

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http://dx.doi.org/10.1016/j.ins.2014.03.073
0020-0255/

where $|E|$ denotes the number of elements of the set E. $\tau_n(A)$ is called the degree of the truth of A in n-valued system.

Definition 2.2 (*Wang and Zhou [1]*). Let $A, B \in F(S)$. Define

$$\xi_n(A,B) = \tau_n((A \to B) \wedge (B \to A)).$$

$\xi_n(A,B)$ is called the degree of similarity between A and B. In the sequel we often write ξ with subscripts to explicitly indicate the logic system involved.

Definition 2.3 (*Wang and Zhou [1]*). Let $A, B \in F(S)$. Define

$$\rho_n(A,B) = 1 - \xi_n(A,B).$$

$\rho_n(A,B)$ is called the pseudo-metric between A and B.

Proposition 2.1 (*Theorem 8(iv) in [1]*).

$\xi_n(A,B) = 0$ if and only if one of A and B is a tautology and the other one is a contradiction. ξ_n here is either $\xi_{Ł_n}$ or $\xi_{R_{0n}}$.

The following counterexample shows that Proposition 2.1 is incorrect.

Example 2.1. (1) In system $Ł_n$, take $A = p^n, B = \neg p^n$, where $p \in S$(the set of all atomic formulas), $p^2 = p\&p, p^{k+1} = p^k\&p, k = 2, 3, \ldots$, and & is defined by $C\&D = \neg(C \to \neg D), C, D \in F(S)$. Since

$$\forall v \in \Omega,\ v(p) \in Ł_n = \left\{0, \frac{1}{n-1}, \ldots, \frac{n-2}{n-1}, 1\right\},$$

$$v(A) = \begin{cases} 1 & v(p) = 1 \\ 0 & v(p) \neq 1 \end{cases}, \quad v(B) = \begin{cases} 0 & v(p) = 1 \\ 1 & v(p) \neq 1 \end{cases},$$

we get, $\forall v \in \Omega,\ v((A \to B) \wedge (B \to A)) = 0$, thus $\tau_n((A \to B) \wedge (B \to A)) = 0$, i.e., $\xi_{Ł_n}(A,B) = 0$. But there is neither a tautology nor a contradiction in A, B.

(2) In system $\mathcal{L}_n^*$, take $A = (\neg p^2)^2, B = \neg A$.

Since

$$\forall v \in \Omega,\ v(p^2) = \begin{cases} 0 & v(p) \leqslant \frac{1}{2} \\ v(p) & v(p) > \frac{1}{2} \end{cases}, \quad v(\neg p^2) = \begin{cases} 1 & v(p) \leqslant \frac{1}{2} \\ 1 - v(p) & v(p) > \frac{1}{2} \end{cases},$$

we get,

$$v(A) = \begin{cases} 1 & v(p) \leqslant \frac{1}{2} \\ 0 & v(p) > \frac{1}{2} \end{cases}, \quad v(B) = \begin{cases} 0 & v(p) \leqslant \frac{1}{2} \\ 1 & v(p) > \frac{1}{2} \end{cases}.$$

Further we get $\forall v \in \Omega,\ v((A \to B) \wedge (B \to A)) = 0$, thus $\tau_n((A \to B) \wedge (B \to A)) = 0$, i.e., $\xi_{R_{0n}}(A,B) = 0$. But there is neither a tautology nor a contradiction in A, B.

Now we correct Proposition 2.1 as follows:

Theorem 2.1. *$\xi_n(A,B) = 0$ if and only if $A \approx \neg B$, and $\forall v \in \Omega,\ v(A) \in \{0,1\}, v(B) \in \{0,1\}$. ξ_n is either $\xi_{Ł_n}$ or $\xi_{R_{0n}}$.*

Proof.

(1) In system $Ł_n$, $\xi_{Ł_n}(A,B) = 0$,
iff $\tau_n((A \to B) \wedge (B \to A)) = 0$,
iff $\forall v \in \Omega,\ v((A \to B) \wedge (B \to A)) = (1 - v(A) + v(B) \wedge (1 - v(B) + v(A)) \wedge 1 = 0$,
iff $\forall v \in \Omega,\ v(A) - v(B) = 1$, or $v(B) - v(A) = 1$,
iff $\forall v \in \Omega,\ v(A) = 1, v(B) = 0$; or $v(A) = 0, v(B) = 1$,
iff $A \approx \neg B$, and $\forall v \in \Omega,\ v(A) \in \{0,1\}, v(B) \in \{0,1\}$.

(2) In system $\mathcal{L}_n^*$, $\xi_{R_{0n}}(A,B) = 0$, iff $\tau_n((A \to B) \wedge (B \to A)) = 0$,
iff $\forall v \in \Omega,\ v((A \to B) \wedge (B \to A)) = (v(A) \to v(B)) \wedge (v(B) \to v(A)) = 0$,
iff $\forall v \in \Omega,\ v(A) \to v(B) = 0$, or $v(B) \to v(A) = 0$,
iff $\forall v \in \Omega, 1 - v(A) \vee v(B) = 0$, or $1 - v(B) \vee v(A) = 0$,
iff $\forall v \in \Omega,\ v(A) = 1, v(B) = 0$; or $v(A) = 0, v(B) = 1$,
iff $A \approx \neg B$, and $\forall v \in \Omega,\ v(A) \in \{0,1\}, v(B) \in \{0,1\}$. □

Since $\rho_n(A,B) = 1$ if and only if $\xi_n(A,B) = 0$, the following proposition is also incorrect.