Information Sciences 278 (2014) 599–613

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Central tendency for symmetric random fuzzy numbers ☆

Beatriz Sinova [a,b,*], María Rosa Casals [a], María Ángeles Gil [a]

[a] Departamento de Estadística, I.O. y D.M., Facultad de Ciencias, Universidad de Oviedo, E-33071 Oviedo, Spain
[b] Department of Applied Mathematics, Computer Science and Statistics, Ghent University, 9000 Gent, Belgium

ARTICLE INFO

Article history:
Received 11 October 2013
Received in revised form 5 February 2014
Accepted 5 March 2014
Available online 27 March 2014

Keywords:
Aumann-type mean of a random fuzzy number
Fuzzy number
L^1 medians of a random fuzzy number
Random fuzzy number
Symmetric fuzzy number
Symmetric random fuzzy number

ABSTRACT

Random fuzzy numbers are becoming a valuable tool to model and handle fuzzy-valued data generated through a random process. Recent studies have been devoted to introduce measures of the central tendency of random fuzzy numbers showing a more robust behaviour than the so-called Aumann-type mean value. This paper aims to deepen in the (rather comparative) analysis of these centrality measures and the Aumann-type mean by examining the situation of symmetric random fuzzy numbers. Similarities and differences with the real-valued case are pointed out and theoretical conclusions are accompanied with some illustrative examples.

1. Introduction

Symmetric random variables sometimes appear exactly in real-life situations, but they mainly correspond to either an idealized or an approximate model for many of them. Symmetric random variables show several interesting properties, especially in connection with their central tendencies. More specifically, the behaviour of the two most popular central tendency measures, the mean and the median, in dealing with symmetric distributions of random variables becomes one of the soundest arguments supporting their adequacy to summarize the central tendency of these variables.

On the other hand, in the last decades fuzzy data have been shown to be a suitable tool in modelling imprecise data coming from judgements/opinions/ratings/valuations/etc. The flexibility of fuzzy numbers allows us to capture the intrinsic imprecision of such data by means of the use of $[0,1]$-valued functions, leading to a powerful and expressive way to 'wording' such ratings/valuations and to an ease-to-develop computation setting.

Random fuzzy numbers, as a special case of the so-coined fuzzy random variables by Puri and Ralescu [19] model a random mechanism generating fuzzy data and extend real-valued random variables (and also random intervals) by allowing data to be fuzzy-valued.

☆ This research has been partially supported by/benefited from the Spanish Ministry of Economy and Competitiveness Grant MTM2009-09440-C02-01, the Principality of Asturias Grant SV-PA-13-ECOEMP-66, and the FPU Grant AP2009-1197 (Sinova) from the Spanish Ministry of Education. Their financial support is gratefully acknowledged.

* Corresponding author at: Departamento de Estadística, I.O. y D.M., Facultad de Ciencias, Universidad de Oviedo, E-33071 Oviedo, Spain. Fax: +34 985103354.

E-mail address: sinovabeatriz@uniovi.es (B. Sinova).

http://dx.doi.org/10.1016/j.ins.2014.03.077
0020-0255/

In Section 2 some preliminaries concerning random fuzzy numbers will be recalled along with convenient extensions of the mean and median values for them. Section 3 introduces and examines the notion of symmetric random fuzzy number about a real value. In Section 4 a discussion is presented on the values these centrality measures take on for symmetric random fuzzy numbers, and the obtained conclusions are compared with those for the real-valued case. In Section 5 a comparative study is developed to examine the proximity of the central tendency measures to 'central position' values of some symmetric random fuzzy numbers. The paper ends with some concluding remarks.

2. Preliminaries

Let $\mathcal{F}_c(\mathbb{R})$ denote the space of fuzzy numbers, where a *fuzzy number* (also called bounded fuzzy number) is a mapping $\widetilde{U}:\mathbb{R}\to[0,1]$ so that for each $\alpha\in[0,1]$ the *α-level set*

$$\widetilde{U}_\alpha=\begin{cases}\{x\in\mathbb{R}:\widetilde{U}(x)\geqslant\alpha\} & \text{if } \alpha>0\\ \text{cl}\{x\in\mathbb{R}:\widetilde{U}(x)>0\} & \text{if } \alpha=0\end{cases}$$

is a nonempty compact interval.

Equivalently, Goetschel and Voxman [15] proved that a fuzzy number is a mapping $\widetilde{U}:\mathbb{R}\to[0,1]$ such that

- $\inf\widetilde{U}_{(\cdot)}:[0,1]\to\mathbb{R}$ is a bounded non-decreasing function,
- $\sup\widetilde{U}_{(\cdot)}:[0,1]\to\mathbb{R}$ is a bounded non-increasing function,
- $\inf\widetilde{U}_1\leqslant\sup\widetilde{U}_1$,
- $\inf\widetilde{U}_{(\cdot)}$ and $\sup\widetilde{U}_{(\cdot)}$ are left-continuous on $(0,1]$ and right-continuous at 0.

When fuzzy data are described by means of elements in $\mathcal{F}_c(\mathbb{R})$, the statistical data analysis involves the usual fuzzy arithmetic based on Zadeh's extension principle [28]. The two key operations, the sum and the product by a scalar, can be equivalently formalized as the level-wise extensions of the usual interval-valued operations, i.e., for $\widetilde{U},\widetilde{V}\in\mathcal{F}_c(\mathbb{R})$ and $\lambda\in\mathbb{R}$, for each $\alpha\in[0,1]$

$$(\widetilde{U}+\widetilde{V})_\alpha=(\text{Minkowski sum of }\widetilde{U}_\alpha\text{ and }\widetilde{V}_\alpha)=\left\{y+z:y\in\widetilde{U}_\alpha,z\in\widetilde{V}_\alpha\right\}$$
$$=\left[\inf\widetilde{U}_\alpha+\inf\widetilde{V}_\alpha,\sup\widetilde{U}_\alpha+\sup\widetilde{V}_\alpha\right],$$
$$(\lambda\cdot\widetilde{U})_\alpha=\lambda\cdot\widetilde{U}_\alpha=\left\{\lambda\cdot y:y\in\widetilde{U}_\alpha\right\}=\begin{cases}\left[\lambda\inf\widetilde{U}_\alpha,\lambda\sup\widetilde{U}_\alpha\right] & \text{if }\lambda\geqslant0\\ \left[\lambda\sup\widetilde{U}_\alpha,\lambda\inf\widetilde{U}_\alpha\right] & \text{otherwise.}\end{cases}$$

As a consequence of this arithmetic, if $\widetilde{U},\widetilde{V}\in\mathcal{F}_c(\mathbb{R})$, then the difference $\widetilde{U}-\widetilde{V}$ can be immediately established by considering $\widetilde{U}-\widetilde{V}=U+(-1)\cdot\widetilde{V}$. At this point, it should be pointed out that $\widetilde{U}-\widetilde{V}+\widetilde{V}\neq\widetilde{U}$. More precisely, $\widetilde{V}-\widetilde{V}\neq\mathbb{1}_{\{0\}}$, but $\widetilde{V}-\widetilde{V}=\mathcal{O}_{\widetilde{V}}$, where for any $\alpha\in[0,1]$ corresponds to the centrally symmetric about 0 interval (see Chakerian [6]) given by

$$\left(\mathcal{O}_{\widetilde{V}}\right)_\alpha=\left[\inf\widetilde{V}_\alpha-\sup\widetilde{V}_\alpha,\sup\widetilde{V}_\alpha-\inf\widetilde{V}_\alpha\right],$$

whence $\mathcal{O}_{\widetilde{V}}$ is a symmetric fuzzy number about 0 which only reduces to $\mathbb{1}_{\{0\}}$ if, and only if, $\widetilde{V}$ reduces to the indicator function of a singleton $\mathbb{1}_{\{v\}}$ ($v\in\mathbb{R}$).

Random elements taking on intrinsic fuzzy number values can be suitably formalized in terms of random fuzzy numbers, a notion which was coined as fuzzy random variable and which was stated in a more general space of fuzzy sets by Puri and Ralescu [19]. The particularization to the case of fuzzy number-valued random elements lead to the following concept:

Definition 2.1. Given a probability space $(\Omega,\mathcal{A},P)$ modelling a random experiment, an associated **random fuzzy number** is a mapping $\mathcal{X}:\Omega\to\mathcal{F}_c(\mathbb{R})$ such that for all $\alpha\in[0,1]$ the interval-valued α-level mapping $\mathcal{X}_\alpha=[\inf\mathcal{X}_\alpha,\sup\mathcal{X}_\alpha]$ is a compact random interval (that is, $\inf\mathcal{X}_\alpha$ and $\sup\mathcal{X}_\alpha$ are two random variables satisfying that $\inf\mathcal{X}_\alpha\leqslant\sup\mathcal{X}_\alpha$).

If $\mathcal{X}:\Omega\to\mathcal{F}_c(\mathbb{R})$ is a random fuzzy number, then one can prove (see Colubi et al. [5]) that it is a Borel-measurable mapping with respect to the Borel σ-field generated on $\mathcal{F}_c(\mathbb{R})$ by the topology associated with several different metrics. The Borel-measurability of random fuzzy numbers allows us to properly establish the *induced distribution of a random fuzzy number*, the *independence of random fuzzy numbers*, and others.

Remark 2.1. It should be emphasized that, although the induced distribution of a random fuzzy number is well-defined, one cannot universally characterize it by means of a distribution function like in the real-valued case. This is due to the fact that there is no ranking for fuzzy numbers which is universally accepted. Indeed, one can define different complete orderings between fuzzy numbers, which show reasonable properties in many cases and being valuable for problems inexcusably requiring a ranking, but none of them can be considered as generally acceptable. For this reason, there is no formal definition