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Evaluating center-seeking and initialization bias: The case of particle swarm and gravitational search algorithms

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ABSTRACT

Complex optimization problems that cannot be solved using exhaustive search require efficient search metaheuristics to find optimal solutions. In practice, metaheuristics suffer from various types of search bias, the understanding of which is of crucial importance, as it is directly pertinent to the problem of making the best possible selection of solvers. In this paper, two metrics are introduced: one for measuring center-seeking bias (CSB) and one for initialization region bias (IRB). The former is based on "ξ-center offset", an alternative to "center offset", which is a common but inadequate approach to analyzing the center-seeking behavior of algorithms, as will be shown. The latter is proposed on the grounds of "region scaling". The introduced metrics are used to evaluate the bias of three algorithms while running on a test bed of optimization problems having their optimal solution at, or near, the center of the search space. The most prominent finding of this paper is considerable CSB and IRB in the gravitational search algorithm (GSA). In addition, a partial solution to the center-seeking and initialization region bias of GSA is proposed by introducing a "mass-dispersed" version of GSA, mdGSA. mdGSA promotes the global search capability of GSA. Its performance is verified using the same mathematical optimization problem, next to a gene regulatory network parameter identification problem. The results of these experiments demonstrate the capabilities of mdGSA in solving real-world optimization problems.

1. Introduction

Consider a *search scenario* in a finite continuous search space $E \subset X$ defined by

$$E = \bigotimes_{d=1}^{D} \left[L_x^d, U_x^d\right], \tag{1}$$

with the objective of locating $\mathbf{x}^* \in E$, where $f(\mathbf{x}^*)$ is the extremum of a function $f(\mathbf{x}) : E \to \mathbb{R}$, and where L_x^d and U_x^d are respectively the lower and upper bound of the search domain at dimension *d*. Optimization problems are to be found in such diverse arenas as engineering, business, medicine, etc. [4]. Here we assume that the only information available to the search

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for the optimal design variable is a measure to discriminate solutions, i.e., for any point $\mathbf{x} \in E$, the associated objective (fitness) value $f(\mathbf{x})$ is assumed to be the only information available to locate $\mathbf{x}^*$. Without loss of generality, a minimization problem is considered.

In contrast to exhaustive search which looks into every entry in the search space, *metaheuristics* [22] are strategies that guide the search process iteratively, in many cases by making a trade-off between exploration and exploitation. This is an important notion when it comes to allocating scarce resources to the exploration of new possibilities and the exploitation of old certainties.

The evolution of life on earth, which has been the original inspiration for many types of metaheuristics, has resulted in the family of population-based stochastic search algorithms termed "evolutionary algorithms". Common to all *population-based* metaheuristics are (i) a measure to discriminate solutions and (ii) a set of mechanisms to modify solutions by various operators.

There are two distinct classes of nature-inspired population-based optimization algorithms that are of our interest: evolutionary algorithms (EA), and swarm intelligence (SI)-based algorithms. Some popular members of the former class are genetic algorithm (GA) [24] and differential evolution (DE) [45,56]. Successful instances of swarm intelligence-based algorithms are particle swarm optimization (PSO) [27] and the gravitational search algorithm (GSA) [46].

Studying the properties of these algorithms, it turns out that some population-based optimization techniques suffer from a specific search bias [11,35]: they tend to perform best when the optimum is located *at or near the center of the search space*. General purpose optimizers are those which make no assumption on the problem at stake. Consequently, if we want to compare the quality of the solutions found by a set of metaheuristics for a series of benchmark problems with optimal solution near the center of the search space, the comparison becomes unfair.

To remedy this unfairness, the so-called *center offset* (CO) [36] approach was proposed which changes the borders of the search space in such a way that the optimal solution is no longer located in the center of the search space. Basically, the CO approach changes the search space of the original problem by reducing it on one side and expanding it at the other. When comparing a set of algorithms *qualitatively*, the comparison is valid since interference tends to be reduced when all the contenders are submitted to the same set of benchmarks, no matter if the shifting has introduced some degree of increase/decrease in the complexity of the search. Our goal, here, is to supplement the comparison by developing *quantitative* measures that can assist the observer in evaluation of the "degree" of CSB of a certain search algorithm. Quantitative measures are succinct and are the preferred disclosure form, not only for (a) a comparison of the degree of CSB in a set of search algorithms, but also when the task is (b) to examine if a single search algorithm has any CSB at all.

On the basis of these observations, we decided to examine generic methods for evaluating the search bias of different algorithms. In this paper, we limit ourselves to two metrics; one for measuring center-seeking bias, and one for initialization bias. These metrics are used to evaluate the behavioral bias of several algorithms related to swarm optimization and gravitational search.

The remainder of this paper is organized as follows. Section 2.1 elaborates on center offset and its assumptions, and presents an alternative. Section 2.2 presents a metric to both measure and compare the center-seeking bias of optimization algorithms. A metric to measure initialization region bias is then presented in Section 3. In Sections 4.1 and 4.2 PSO and GSA are briefly summarized. The mass assignment in GSA is analyzed and challenged in Section 4.3, and an alternative is proposed. The experimental setup adopted for the evaluation and comparison, followed by the major observations derived form the experiment, are presented in Section 5. Section 6 presents discussions and provides a framework that enables a fair comparison of optimization heuristics. The last section highlights conclusions and provides suggestions for future research.

2. A metric for measuring center-seeking bias

2.1. Understanding the assumptions underlying center offset

According to the *No Free Lunch* theorem [60], all learning systems will expose equal performance over all possible cost functions. This implies that, in order to efficiently solve an optimization problem, they should be tailored to the salient problem-specific characteristics. Where there is no available information on the problem at hand, as with various real-world applications, some search biases known to us are not often of service. Such biases include center-seeking (CS) behavior and initialization region bias (IRB), the foci of this study.

When comparing nature-inspired metaheuristic algorithms, a symmetric search space can be misleading when the optimal solution is located at, or near, the center of the search space. In such a case, one must account for CS behavior in order to draw valid conclusions from an experiment [7]. One attempt to deal with CS bias is called *center offset* (CO). This is a common approach to negating the centrist bias of an optimization algorithm [3]. The underlying assumption of CO is that the complexity of a problem does not change as a result of moving the optimal solution from the center of the search space; this is an assumption that is discussed in greater detail below.

When applying CO, the optimization problem $f(\mathbf{x})$ is changed to $f(\mathbf{x} - \mathbf{C})$ where $\mathbf{C}$ is the location of the new center. CO is equivalent to expanding the search space from one side, for each dimension d, and to shrinking it on the other side, without changing the distance $\left\| U_x^d - L_x^d \right\|$ between the lower bound L_x^d and the upper bound U_x^d. When the objective of a test is to measure the search bias of an algorithm, CO is not an adequate approach. This is because a change in the complexity of a problem