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# Chebyshev type inequalities for general fuzzy integrals

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ABSTRACT

In this article, we first introduce a class of binary operation called $g$-seminorm, which generalizes the concept of $t$-seminorm. Then we use the $g$-seminorm to define a class of Sugeno-like integral. Finally, we establish some new Chebyshev type inequalities for this kind of integral.



## 1. Introduction

The fuzzy integral with respect to a fuzzy measure was first proposed by Sugeno [16]. The concept of semi(co)normed fuzzy integral, which is the extension of Sugeno's fuzzy integral, was introduced by Suarez and Gil [17]. From a mathematical point of view, fuzzy integral has very interesting properties that have been studied by many authors, including Ralescu and Adams [11], Román-Flores et al. [13,14] and Wang and Klir [18] among others.

The study of inequalities for the fuzzy integral was initiated by Román-Flores et al. [7], and then followed by the others [1–3,5,9]. Recently, several authors [4,10] have studied some fuzzy integral inequalities involving $t$-seminorm. Motivated by these results, in this article, we first create a general fuzzy integrals, and then we establish some new Chebyshev type inequalities for the fuzzy integral. Our results extend many special inequalities which have appeared in the recent literature to general cases.

Firstly, we recall some basic notations and definitions.

Let $X$ be a non-empty set, and $\Sigma$ a $\sigma$-algebra of subsets of $X$. Let $\mu : \Sigma \to B$ (where $B = [0,1]$ or $B = [0,\infty]$ or $B = [0,\infty)$) be a set function. We say that $\mu$ is a fuzzy measure if it satisfies

(1) $\mu(\phi) = 0$ and $\mu(X) > 0$.
(2) $E, F \in \Sigma$ and $E \subset F$ imply $\mu(E) \leqslant \mu(F)$.
(3) $E_n, E \in \Sigma, E_n \to E$ imply $\mu(E_n) \to \mu(E)$.

When $\mu$ is a fuzzy measure, the triple $(X, \Sigma, \mu)$ is called a fuzzy measure space.

Let $(X, \Sigma, \mu)$ be a fuzzy measure space. The symbol $\mathcal{F}^\mu(X)$ denotes the set of all nonnegative measurable functions with respect to $\Sigma$.

In what follows, all considered functions belong to $\mathcal{F}^\mu(X)$, and for any $\alpha \geqslant 0$, the set $\{x \in X | f(x) \geqslant \alpha\}$ is denoted by $F_\alpha$.

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**Definition 1.1** [16]. Let $(X, \Sigma, \mu)$ be a fuzzy measure space, $f \in \mathcal{F}^{\mu}(X)$ and $A \in \Sigma$, then the Sugeno integral (or fuzzy integral) of $f$ on $A$ with respect to the fuzzy measure $\mu$ is defined by

$$(s)\int_A f d\mu = \bigvee_{\alpha \in B} [\alpha \wedge \mu(A \cap F_\alpha)],$$

where $\vee$ and $\wedge$ denote the operations sup and inf on $B$, respectively.

**Definition 1.2** [17]. A $t$-seminorm is a function $T : [0,1] \times [0,1] \to [0,1]$ which satisfies

(1) $T(x,1) = T(1,x) = x$ for each $x \in [0,1]$.
(2) If $x_1 \leqslant x_3, x_2 \leqslant x_4$ for each $x_1, x_2, x_3, x_4 \in [0,1]$, then $T(x_1, x_2) \leqslant T(x_3, x_4)$.

**Definition 1.3** [17]. Let $T$ be a $t$-seminorm, then the seminormed fuzzy integral of $f$ over $A$ with respect to $T$ and the fuzzy measure $\mu$ is defined by

$$\int_{TA} f d\mu = \bigvee_{\alpha \in [0,1]} T[\alpha, \mu(A \cap F_\alpha)].$$

Obviously, the seminormed fuzzy integral is the fuzzy integral for the case $T(x,y) = x \wedge y$ and $B = [0,1]$.

**Definition 1.4** [12]. Two functions $f, g : X \to B$ are said to be comonotone if for all $(x,y) \in X^2, (f(x) - f(y))(g(x) - g(y)) \geqslant 0$. A family $\{f_i\}_{i=1}^n$ of measurable functions is a system of comonotone functions if and only if each pair of these functions is comonotone.

## 2. Main results

In this section, we first consider the following general fuzzy integral that can unify Definitions 1.1 and 1.3.

**Definition 2.1.** A function $G : B \times B \to B$ is called a generalized seminorm ($g$-seminorm for short) if it satisfies

(1) $G(x, \mu(X)) \leqslant x$ for each $x \in B$.
(2) If $x_1 \leqslant x_3, x_2 \leqslant x_4$ for each $x_1, x_2, x_3, x_4 \in B$, then $G(x_1, x_2) \leqslant G(x_3, x_4)$.

**Remark 2.1.** Note that the $g$-seminorm is a real generalization of $t$-seminorm. For example, for $B = [0,1]$ and $\mu(X) = 1$, if we take $G(x,y) = (x^2)y$, then $G$ is a $g$-seminorm but not a $t$-seminorm. Also, for any $B$, if $\mu(\Sigma) \subset B$ and $\mu(X) < \infty$, then $G(x,y) = xe^{\frac{1}{2}y - \mu(X)}$ is a $g$-seminorm but not a $t$-seminorm.

**Definition 2.2.** Let $G$ be a $g$-seminorm. Define the $g$-seminorm fuzzy integral of $f$ on a set $A \in \Sigma$ with $\mu$ as

$$\int_{GA} f d\mu = \bigvee_{\alpha \in B} G(\alpha, \mu(A \cap F_\alpha)).$$

**Remark 2.2.** If take $G = T$ with $B = [0,1]$ in Definition 2.2, then we obtain the seminormed fuzzy integral in Definition 1.3. Also, if take $G = \wedge$, then we obtain the Sugeno integral.

Now we state and prove the following Chebyshev type inequalities for the $g$-seminorm fuzzy integral, which is our main assertion.

**Theorem 2.1.** *Let $H : B^n \to B$ be a left continuous and non-decreasing $n$-place function. Let $\varphi : B \to B$ be any strictly monotone increasing bijection, and suppose that $f_1, \cdots, f_n : X \to B$ is any comonotone system. If the $g$-seminorm $G$ satisfies*

$$G(\varphi(H(x_1, \ldots, x_n)), c) \geqslant max \begin{Bmatrix} H(G(\varphi(x_1), c), \varphi(x_2), \ldots, \varphi(x_n)), \\ H(\varphi(x_1), G(\varphi(x_2), c), \ldots, \varphi(x_n)), \\ \cdots \\ H(\varphi(x_1), \ldots, \varphi(x_{n-1}), G(\varphi(x_n), c)). \end{Bmatrix} \quad (1)$$