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Modeling and forecasting financial time series with ordered fuzzy candlesticks

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ABSTRACT

The goal of the paper is to present an experimental evaluation of fuzzy time series models which are based on ordered fuzzy numbers to predict financial time series. Considering this approach the financial data is modeled using Ordered Fuzzy Numbers (OFNs) called further by Ordered Fuzzy Candlesticks (OFCs). The use of them allows modeling uncertainty associated with financial data and maintaining more information about price movement at assumed time interval than comparing to commonly used price charts (e.g. Japanese Candlestick chart). Thanks to well-defined arithmetic of OFN, one can construct models of fuzzy time series, such as an Ordered Fuzzy Autoregressive Process (OFAR), where all input values are OFC, while the coefficients and output values are arbitrary OFN; in the form of classical equations, without using rule-based systems. In an empirical study ordered fuzzy autoregressive models are applied to modeling and predict price movement of futures contracts on Warsaw Stock Exchange Top 20 Index.

1. Introduction

Lo and Mackinlay consider that the financial market is a complex, non-stationary, noisy, chaotic and dynamic system but it does not follow random walk [13]. The main reason is the fact that a huge amount of information is reflected on the financial market. Main factors include an economic condition, political situation, traders' expectations, catastrophes and other unexpected events. So one can conclude that stock market data should be considered in the framework of uncertainties. Therefore, predictions of stock market price and their directions are quite difficult.

Moreover, modern financial data sets may contain tens of thousands of quotes in a single day time stamped to the nearest second. Making investment decisions based on observation of each single quotation is very difficult or even impossible. Therefore a large part of investors very often use price charts analysis to make decisions. The price charts (e.g. Japanese Candlestick chart) are used to illustrate movements in the price of a financial instrument over time. Notice, that using the price chart, a large part of the information about the process is lost, e.g. using Japanese Candlestick chart with daily frequency, for one day, we know only four prices (i.e. open, low, high and close), while in this time the price has changed hundreds of times. In spite of this Japanese Candlestick charting techniques are very popular among traders and allow for achieve more than average profits. More details about the Japanese Candlesticks and trading techniques based on them can be found in [17].

This paper is a continuation and extension of our previous works [14,15] bringing several new results and numerical examples. In the presented approach was proposed how with ordered fuzzy numbers, one can modeling uncertainty

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associated with financial data and reduce the size of lost information. Further, using a concept of the ordered fuzzy candlestick, the models of financial time series are built. Next, those models are applied to modeling and to predict price movement of futures contracts on Warsaw Stock Exchange Top 20 Index (WIG20).

2. Methodology of ordered fuzzy models

2.1. Ordered Fuzzy Numbers (OFNs)

Ordered fuzzy numbers introduced by Kosiński et al. in series of papers [4–9] are defined by ordered pairs of continuous real functions defined on the interval [0,1] i.e. $A = (f, g)$ with $f, g : [0,1] \to \mathbb{R}$ as continuous functions.

Functions f and g are called the *up* and *down*-parts of the fuzzy number A, respectively. The continuity of both parts implies their images are bounded intervals, say *UP* and *DOWN*, respectively. In general, the functions f and g need not be invertible, and only continuity is required. If we assume, however, that these functions are monotonous, i.e., invertible, and add the constant function of x on the interval $[1_A^-, 1_A^+]$ with the value equal to 1, we might define the membership function

$$\mu(x) = \begin{cases} f^{-1}(x) & \text{if} \quad x \in [f(0), f(1)], \\ g^{-1}(x) & \text{if} \quad x \in [g(1), g(0)], \\ 1 & \text{if} \quad x \in [1_A^-, 1_A^+], \end{cases} \tag{1}$$

if f is increasing and g is decreasing, and such that $f \leqslant g$ (pointwise). In this way, the obtained membership function $\mu(x)$, $x \in \mathbb{R}$ represents a mathematical object which resembles a convex fuzzy number in the classical sense [2,11,12,19,20]. The ordered fuzzy number and ordered fuzzy number as a fuzzy number in classical meaning are presented in Fig. 1.

Furthermore, the basic arithmetic operations on ordered fuzzy numbers are defined as the pairwise operations of their elements.

Let $A = (f_A, g_A)$, $B = (f_B, g_B)$ and $C = (f_C, g_C)$ are ordered fuzzy numbers. The sum $C = A + B$, subtraction $C = A - B$, product $C = A \cdot B$, and division $C = A \div B$ are defined by formula

$$f_C(y) = f_A(y) * f_B(y), \quad g_C(y) = g_A(y) * g_B(y) \tag{2}$$

where $*$ works for $+$, $-$, $\cdot$ and $\div$, respectively, and where $C = A \div B$ is defined, if the functions $| f_B |$ and $| g_B |$ are bigger than zero. In a similar way, multiply an ordered fuzzy number A by a scalar $\lambda \in \mathbb{R}$, i.e. $C = \lambda \cdot A$ is defined by formula

$$f_C(y) = \lambda \cdot f_A(y), \qquad g_C(y) = \lambda \cdot g_A(y) \tag{3}$$

This definition leads to some useful properties. The one of them is existence of neutral elements of addition and multiplication. This fact causes that not always the result of an arithmetic operation is a fuzzy number with a larger support. This allows to build fuzzy models based on ordered fuzzy numbers in the form of the classical equations without losing the accuracy.

Moreover, a universe $\mathcal{O}$ of all ordered fuzzy numbers can be identified with $C^0([0,1]) \times C^0([0,1])$, hence the space $\mathcal{O}$ is topologically a Banach space [8]. A class of defuzzification operators of ordered fuzzy numbers can be defined, as linear and continuous functionals on the Banach space $\mathcal{O}$. Each of them, say *Def* has a representation by a sum of two Stieltjes integrals with respect to functions v_1 and v_2 of bounded variation [10],

$$Def(A) = \int_0^1 f_A(s)dv_1(s) + \int_0^1 g_A(s)dv_2(s) \tag{4}$$

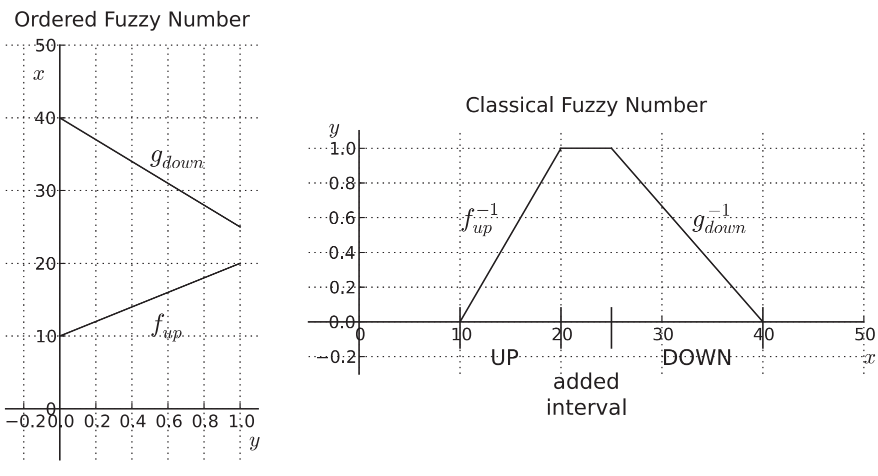

Fig. 1. Graphical interpretation of OFN and an OFN presented as fuzzy number in classical meaning.