Information Sciences 188 (2012) 253–259

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

# Diagnosability of star graphs with missing edges ☆

Chieh-Feng Chiang [a], Guo-Huang Hsu [b], Lun-Min Shih [c], Jimmy J.M. Tan [a,]*

[a] *Department of Computer Science, National Chiao Tung University, Hsinchu 30050, Taiwan*
[b] *Industrial Technology Research Institute, Hsinchu City, Taiwan*
[c] *Department of Continuing and Extension Education, Da Yeh University, Taiwan*

ARTICLE INFO

*Article history:*
Received 25 August 2008
Received in revised form 27 July 2011
Accepted 7 November 2011
Available online 16 November 2011

*Keywords:*
Star graph
Comparison diagnosis model
MM* diagnosis model
Local diagnosability
Extended star structure
Strong local diagnosability property

ABSTRACT

In this paper, we study the system diagnosis on an $n$-dimensional star under the comparison model. Following the concept of local diagnosability [3], the strong local diagnosability property [7] is discussed; this property describes the equivalence of the local diagnosability of a node and its degree. We prove that an $n$-dimensional star has this property, and it keeps this strong property even if there exist $n-3$ missing edges in it.


## 1. Introduction

In recent years, with the continuing advancements in semiconductor technology, large multiprocessor systems such as very-large-scale integration (VLSI) systems have become increasingly popular. Such systems must be capable of uninterrupted processing, and therefore, the reliability of the processors in these systems should be considered. The diagnosis of such systems involves the identification of all the faulty processors in the system. The diagnosability of the system refers to the maximum number of faulty processors that can definitely be identified.

Several approaches to system diagnosis have been developed in previous researches. One major approach, called the comparison diagnosis model, was proposed by Maeng and Malek [13,14]. In this model, diagnosis is performed by simultaneously sending two identical signals from a processor to two other linked processors and then comparing the responses. The test results are collected and analyzed to identify all the faulty processors. Following the traditional concept of diagnosability, many variants of diagnosability measurements have been presented. A different measurement called conditional diagnosability was proposed [8], and a more precise concept called strong diagnosability has been widely applied to various networks [4–6].

In contrast to the traditional concept of diagnosability, Chiang and Tan [3] introduced a different concept for system diagnosis called local diagnosability; this method requires only the correct identification of the status of a single processor. Each processor has its own local diagnosability, and there exists a strong relationship between the local diagnosability and the traditional diagnosability. In the comparison diagnosis model for a given processor, a local structure called an extended star has also been presented for guaranteeing a processor's local diagnosability.

☆ This work was supported in part by the National Science Council of the Republic of China under Contract NSC 99-2221-E-009-083-MY3.
* Corresponding author. Tel.: +886 3 5715900; fax: +886 3 5721490.
*E-mail address:* jmtan@cs.nctu.edu.tw (J.J.M. Tan).

0020-0255/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2011.11.012

Among all well-known topologies, the star graph is one of the most popular ones. Its features include node symmetry, edge symmetry, regular and low degree of node, and small diameter. Since its introduction, this topology has attracted considerable attention. Some studies have discussed the diameter and fault diameters [9,15,16]. When linearly many vertices are deleted in a star graph, the resulting graph has a large connected component containing almost all remaining vertices [2]. The problem of embedding a linear array of vertices (or a ring) into the star graph has also been solved, even when there exist some vertex faults in the target star graph [10]. With edge faults, the star graph has been proved to have fault-tolerant Hamiltonian laceability [12]. The robustness of star graphs with edge faults has been addressed [11], and the improvement of bounds on edge failure tolerance has also been investigated [18]. For system diagnosis, Zheng et al. [20] showed that the traditional diagnosability of an $n$-dimensional star is $n-1$ for $n \geqslant 4$.

In this paper, we study system diagnosis by following the concept of local diagnosability [3]. Based on this concept, we obtain a simple proof of the fact that the diagnosability of an $n$-dimensional star $S_n$ is $n-1$ for $n \geqslant 4$; this is the same result as that obtained by Zheng et al. [20]. Moreover, we study the diagnosability of a star graph in the presence of arbitrary distributed missing edges under the comparison diagnosis model. A relative study was discussed for the case of a hypercube by Wang [19]. Furthermore, we have studied the strong local diagnosability property [7]. A given processor has the strong local diagnosability property if its local diagnosability equals its degree, where the degree is defined as the number of links incident to this processor. A system has the strong local diagnosability property if every processor in it has this property. We prove that each processor in an $n$-dimensional star $S_n$ has this strong local diagnosability property, and this property is maintained even if $S_n$ has up to $n-3$ missing edges. The number $n-3$ is tight in the sense that the strong local diagnosability property cannot be guaranteed if there are $n-2$ missing edges.

The remainder of this paper is organized as follows. In Section 2, we present some definitions, notations, and terminologies. The concept of local diagnosability for system diagnosis is also introduced in this section. Then, in Section 3, we prove that an $n$-dimensional star keeps the strong local diagnosability property even if there exist $n-3$ missing edges in it. Finally, some conclusions are presented in Section 4.

## 2. Preliminaries and local diagnosability

The topology of a multiprocessor system can be modeled as an undirected graph $G = (V,E)$, where the set of nodes $V$ represents the set of all processors and the set of edges $E$ represents the set of all connecting links between the processors. Let $G'$ be a subgraph of $G$ and $v$ be a node in $G'$; then, $deg_{G'}(v)$ denotes the degree of $v$ in subgraph $G'$. The neighborhood set of a node $v$, denoted by $N(v)$, is defined as the set of all nodes adjacent to $v$.

Let $n$ be a positive integer and $\langle n\rangle$ be the set $\{1, 2\ldots n\}$. An $n$-dimensional star [1], denoted by $S_n$, is a graph whose set of nodes consists of all permutations on $\langle n\rangle$. Each node is uniquely assigned a label $x_1x_2\ldots x_n$, where $x_i \in \langle n\rangle$ for $1 \leqslant i \leqslant n$ and $x_i \neq x_j$ for $i \neq j$. Each node $x_1x_2\ldots x_{i-1}x_ix_{i+1}\ldots x_n$ is adjacent to the nodes $x_ix_2\ldots x_{i-1}x_1x_{i+1}\ldots x_n$ for $2 \leqslant i \leqslant n$, that is, nodes obtained by the transposition of the first coordinate with the $i$th coordinate of the node. Consequently, there exist $n!$ nodes in an $n$-dimensional star, and each node has degree $n-1$. Let $\mathbf{x} = x_1x_2\ldots x_n$ be a node in an $n$-dimensional star $S_n$. We use $(\mathbf{x})_i$ to denote the $i$th coordinate $x_i$ of $\mathbf{x}$ for $1 \leqslant i \leqslant n$. We say that two nodes $\mathbf{x}$ and $\mathbf{y}$ in $S_n$ are adjacent to each other with an $i$th edge or an edge in dimension $i$ if $\mathbf{x}$ can be obtained by the transposition of the first coordinate with the $i$th coordinate of $\mathbf{y}$. Then, $\mathbf{x}$ is said to be the $i$th neighbor of $\mathbf{y}$ and it is denoted as $\mathbf{x} = \mathbf{y}^i$, and vice versa. In addition, we use $S_n^i$ to denote the subgraph of $S_n$ that is induced by the nodes $\mathbf{x}'s$ with $(\mathbf{x})_n = i$ for $1 \leqslant i \leqslant n$. Thus, $S_n$ can be decomposed into $n$ subgraphs $S_n^i$ for $1 \leqslant i \leqslant n$ and each $S_n^i$ is isomorphic to $S_{n-1}$.

Under the comparison model [13,14], a system performs diagnosis by the specific procedure described below. For each processor $w$ linked to two distinct processors $u$ and $v$, the diagnosis is performed by simultaneously sending two identical signals from $w$ to $u$ and from $w$ to $v$, and then comparing their returning responses. The comparison result of $w$ for the two responses from $u$ and $v$ is denoted by $r((u,v)_w)$. An agreement is denoted by $r((u,v)_w) = 0$, whereas a disagreement is denoted by $r((u,v)_w) = 1$. Because the comparator processor might be faulty, if $r((u,v)_w) = 1$, at least one member of $\{u,v,w\}$ is faulty; or, if $r((u,v)_w) = 0$ and $w$ is known to be fault-free, both $u$ and $v$ are fault-free. Furthermore, a special case of the comparison model, called the MM* model [17], assumes that a comparison is performed by each processor for each pair of distinct connected neighbors.

A labeled multigraph $M = (V,C)$, called a comparison graph, is usually used to model this diagnosis strategy, where $V$ represents the set of all processors in $G$ and $C$ represents the set of labeled edges. Each labeled edge $(u,v)_w \in C$ implies that processors $u$ and $v$ are being compared by processor $w$.

The collection of all test results of a test assignment is called a syndrome. Formally, a syndrome is a function $\sigma{:}C \to \{0,1\}$. For a given syndrome $\sigma$, a subset of processors $F \subset V(G)$ is said to be consistent with $\sigma$ if the syndrome $\sigma$ can be produced when all processors in $F$ are faulty and all processors in $V-F$ are fault-free. Let $\sigma_F$ denote the set of syndromes that are consistent with $F$. We say that a system is diagnosable if for every syndrome $\sigma$, a unique set of processors $F \subset V$ is consistent with it. A system is defined to be $t$-diagnosable if the system is diagnosable as long as the number of faulty processors does not exceed $t$. In other words, a system is $t$-diagnosable if given a test syndrome $\sigma_F$ produced by the system under the presence of a set of faulty nodes $F$ with $|F| \leqslant t$, any set of faulty nodes $F'$ consistent with $\sigma_F$ with $|F'| \leqslant t$ must be $F' = F$. The maximum number $t$ for which a system is $t$-diagnosable is called the diagnosability of the system. Two distinct subsets of processors $F_1, F_2 \subset V$ are distinguishable if and only if every syndrome consistent with $F_1$ differs from that consistent with $F_2$.