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Edge fault tolerance of super edge connectivity for three families of interconnection networks

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ABSTRACT

Let $G = (V,E)$ be a connected graph. G is said to be super edge connected (or super-λ for short) if every minimum edge cut of G isolates one of the vertex of G. A graph G is called m-super-λ if for any edge set $S \subseteq E(G)$ with $|S| \leqslant m$, $G - S$ is still super-λ. The maximum cardinality of m-super-λ is called the edge fault tolerance of super edge connectivity of G. In this paper, we discuss the edge fault tolerance of super edge connectivity of three families of interconnection networks.

1. Introduction

We use Bondy and Murty [3] for terminology and notation not defined here and only consider finite simple undirected graphs. Let $G = (V,E)$ be a connected graph. For $v \in V(G)$, the degree of v, written by $d(v)$, is the number of edges incident with v. Let $\delta(G) = \min\{d(v)|v \in V(G)\}$ and it is called the *minimum degree* of G. For a subset S of $V(G)$, $G[S]$ is the subgraph of G induced by S. An edge subset $T \subseteq E(G)$ is an *edge cut* if $G - T$ is disconnected. The *edge connectivity*, denoted by λ, is the minimum cardinality of the set of all edge cuts of G.

It is well known that the edge connectivity λ is an important measurement for the fault tolerance of networks. In general, the larger λ is, the more reliable a network is. Obviously, $\lambda \leqslant \delta(G)$. In [2], Bauer et al.*** defined the so-called super-λ graphs. A graph G is said to be *super edge-connected* (in short, *super-λ*) if every minimum edge cut is the set of edges incident with some vertex of G. There are much research on super-λ, the reader is referred to [5,11,13,16,18] and the references therein.

In [8,9], Esfahanian and Hakimi proposed the concept of restricted edge connectivity of graphs which generalized the concept of super-λ. Then Fábrega and Fiol [10] introduced the k-restricted edge connectivity of interconnection networks. Let G be a graph. An edge set $S \subset E$ is said to be a k-restricted edge cut if $G - S$ is disconnected and there are no components whose cardinalities are smaller than k in $G - S$. The minimum cardinality of k-restricted edge cut of G is called k-restricted edge connectivity of G, denoted by $\lambda_k(G)$. k-restricted edge connectivity is another important parameter in measuring the reliability and fault tolerance of large interconnection networks. In particular, estimating the bound for $\lambda_k(G)$ is of great interest, and many results have been obtained in [1,6,12,17,19–26].

In [14], Hong and Meng defined another index to measure the reliability of networks.

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Definition 1.1 ([14]). A graph G is said to be *m-super edge connected* (*m-super-λ* for short) if $G - S$ is *super-λ* for any $S \subseteq E(G)$ with $|S| \leqslant m$.

From the definition, we know that G is 0-super-λ is equivalent to that G is super-λ. Furthermore, if G is a-super-λ, then G is also b-super-λ, for any $0 \leqslant b \leqslant a$. So m-super-λ is a generalization of super-λ.

The *edge fault tolerance of super edge connectivity* of G is an integer m such that G is *m-super-λ* but not $(m+1)$*-super-λ*, denoted by $S_\lambda(G)$.

In [14], Hong and Meng gave an upper and lower bound for $S_\lambda(G)$. Moreover, more refined bounds for $S_\lambda(G)$ of Cartesian product graphs, edge transitive graphs and regular graphs are given.

In this paper, we will give some bounds of $S_\lambda(G)$ for three families of interconnection networks.

Before proceeding, we introduce some notions which will be used in the discussions in the next sections. Let $G = (V,E)$ be a graph. For two disjoint vertex sets $U_1, U_2 \subseteq V(G)$, we use $[U_1,U_2]_G$ to denote the edge set of G with one end in U_1 and the other end in U_2. For any vertex set $A \subseteq V(G)$, denote $\omega_G(A) = \left|\left[A,\overline{A}\right]_G\right|$, where $\overline{A} = V(G) - A$ is the complement of A. The subscription G is omitted when the graph under consideration is obvious. Next we cite two lemmas which will be used in the following proofs.

Lemma 1.2 ([14]). *A graph G is super-λ if and only if $\omega(A) > \delta(G)$ for any $A \subset V(G)$ with $2 \leqslant |A| \leqslant \left\lfloor \frac{|V(G)|}{2} \right\rfloor$ and $G[A]$ and $G[\overline{A}]$ being connected.*

Lemma 1.3 ([14]). *Let G be a connected graph with minimum degree $\delta(G)$. Then $S_\lambda(G) \leqslant \delta(G) - 1$.*

2. Three families of interconnection networks

The following three families of interconnection networks which we will discuss in the next sections were introduced in [4].

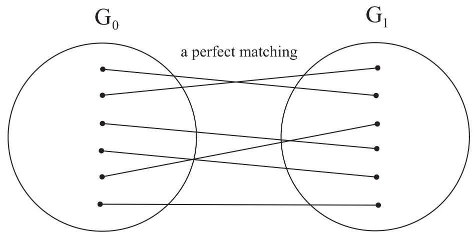

Fig. 1. Graph $G(G_0, G_1; M)$.

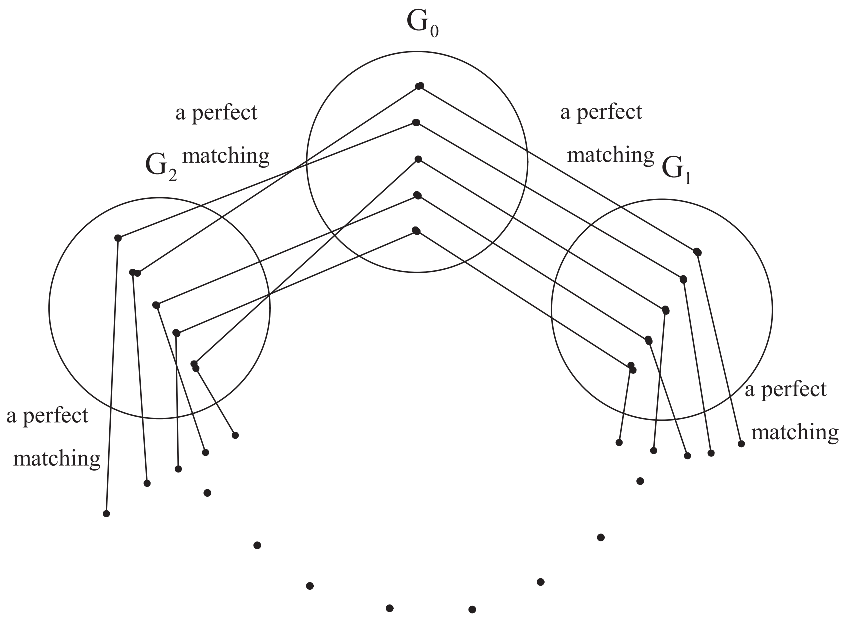

Fig. 2. Graph $G\left(G_0, G_1, \ldots, G_{r-1}; \widetilde{M}\right)$.