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A novel approach to probability distribution aggregation

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ABSTRACT

Today's business world is highly competitive and unpredictable, so effective decision-making is of primary importance. However, it is difficult to make effective decisions when sufficient information is not available, and decision-making in such situations involves a high risk of error. Conventional statistics based approaches to such problems are not effective, because in such situations decision-making is usually in the hands of a small panel of experts. However, the expert opinions can be represented by probability distribution functions. Thus, such a problem reduces to the aggregation of a set of probability distribution functions to an aggregated or consensus distribution. In this paper, we propose a new approach to address this problem. The novelties of the proposed approach include: (1) the problem is formulated as an optimization problem and (2) the overlapping area between an individual expert's distribution and an aggregated distribution is taken to measure the expertise level of that expert and subsequently to determine the weight of the expert. The proposed approach in this paper is illustrated by an example reported in literature handled with the Delphi method, which also shows the effectiveness of our approach.

1. Introduction

In many management and control problems, a decision must be made at a point in time where adequate information for forming the decision is not available [7,21,27,30,31]. For instance, in emergency management, especially at the initialization stage of a project, one may need to estimate the goal of the entire event, a difficult task because of a lack of adequate information at such an early stage. It is generally true that in such situations, where a decision target is highly uncertain, decision-makers or experts may take conventional statistics-based approaches to deal with the problem.

Because different decision-makers inherently have different judgments or opinions on a decision target, aggregation of those different opinions is formed by a consensus [8,10]. The decision-maker's opinion can be represented by a probability distribution function (PDF) [23,33,34,37]. Thus, the problem reduces to the aggregation of a set of distributions to an aggregated or consensus distribution. The problem can be defined mathematically as follows: Denote an aggregated probability distribution function (PDF) as $f(x)$ and its cumulative density function (CDF) as $F(x)$. Note that $F(x)$ can be derived from $f(x)$. Then, we express $F(x)$ as a linear combination of individual decision-makers' CDFs $F_i(x)$, i.e.,

$$F(x) = \sum_{i=1}^{m} w_i F_i(x), \tag{1}$$

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where w_i is the weight of individual decision-maker ($0 \leqslant w_i \leqslant 1$ and $\sum_{i=1}^{m} w_i$). It should be noted that Eq. (1) is also called the weighted average model, which is the simplest expression to represent $F(x)$ in terms of $F_i(x)$. $F(x)$ may be expressed as some non-linear function of $F_i(x)$, which is out of the scope of the present paper.

The objective of the study presented in this paper is to develop a method to determine the weights in Eq. (1). In Section 2, we will provide some background knowledge as well as related work reported in literature. Section 3 comprises our methodology and in Section 4 we present an example that illustrates and verifies our method. Section 5 comprises discussion on our methodology. A concluding remark is provided in Section 6.

2. Background and related work

The ultimate criterion for determining weights is that experts obtain a "fair" or "average" decision (on an event). "Fairness" implies the following proposition that a "better" decision-maker should be assigned a "higher" weight. The issue thus is reduced to two questions: (1) What should be criterion applied to evaluate decision-makers? and (2) how should weight be assigned to decision-makers?

The answer to these questions has been extensively studied in the literature. One method is to assign weight based on the ranking of the decision-maker for the event upon which the decision is to be made [37]. For example, m decision-makers are providing their opinions, and the ith decision-maker has rank r_i. The weight for that decision-maker is given by Winkler [37]:

$$w_i = \frac{r_i}{\sum_{i=1}^{m} r_i}. \tag{2}$$

In the literature, many methods have been proposed to determine the ranking of a decision-maker. The Delphi method is a popular tool for determining the rankings of decision-makers in a group [9,28]. The analytic hierarchy process (AHP) technique is another powerful tool for this purpose [20]. In a particular application, Lai et al. have shown that the AHP is preferable to the Delphi method, as the AHP approach seems to help group members to concentrate the discussion on objectives rather than on alternatives [20].

Fuzzy set theory has been combined with AHP to evaluate the performance of manufacturing systems including factors of cost, flexibility, quality, speed, and dependability [13]. Further, fuzzy logic has been proposed for dealing with vagueness in the subjective evaluation of data and has been applied to principal component analysis and correspondence analysis [24]. An algorithm has been presented for clustering data sets based on the property of the aggregation of pheromones ("Pheromones" are chemicals capable of acting outside the body of the secreting individual to impact the behavior of the receiving individual) found in ants [12]. In particular, as reported in [12], the movement of an ant is governed by the amount of pheromone deposited at different points of a search space, and the aggregation is greater if the deposition of pheromone is greater – leading to the formation of homogenous groups of data [12].

Pasi and Yager proposed linguistic quantifiers associated with aggregation operators to compute a majority opinion by aggregating the individual opinions [29]. In their work, the majority opinion corresponds to the aggregated value, and an ordered weighted averaging (OWA) operator was used to model the semantics of linguistic quantifiers. They also proposed the formalization of a fuzzy majority opinion as a fuzzy subset based on the fact that "majority" is a vague concept. Wang proposed that the weights of the OWA operator be used to aggregate preference rankings, which allowed the weights to be associated with different rankings determined in terms of a decision-makers' optimism level [36]. An attempt was made in [39] to develop a comprehensive theory of information aggregation by means of a penalty function to help the aggregation process.

A method was developed in [4] for the identification of public knowledge and the elimination of biases in this knowledge when information is aggregated in small-group settings [4]. In this method, the non-linear aggregation of an individual's decisions calculates the probability of the future outcome of an uncertain event, which can then be compared to both objectives: probability of its occurrence and the performance of the market. Choi introduced a situation assessment algorithm to reflect a decision situation in the aggregation process [6]. This algorithm includes the factor of cultural importance in a group-decision support system. The model for the group-ranking problem was based on measurement of the degree of preference [17]. This model removes assumptions of certain beliefs in pair-wise rankings, homogeneity that implies equal expertise of all decision-makers with respect to all evaluations, and a full list of requirements according to which each decision-maker evaluates and ranks all alternatives.

A decision model in the form of a recursive aggregation algorithm was developed to mimic a multi-step ranking process of a set of alternatives in a multi-criteria and multi-expert decision-making environment [35]. The concept of relative strength and weakness was used for the comparison of the measures of the distance between any two probability distributions. This approach was found to be suitable for the selection of distances as quasi-optimal design criteria in problems such as signal selection and detector design when the preferred criteria, probability of error, and asymptotic relative efficiency are inflexible [1]. Mesiar et al. discussed a minimization-based aggregation operator by introducing both a weighting function and a dissimilarity function [22]. Group decision-making problems with multiple types of linguistic preference relations were investigated in [38]. In that paper, uncertain additive linguistic preference relations are transformed into expected additive linguistic preference relations and then developed into a method for reaching consensus among individual preferences and a group's opinion.